



I L L I N O I S

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

-

PRODUCTION NOTE

University of Illinois at
Urbana-Champaign Library
Large-scale Digitization Project, 2007.

UNIVERSITY OF ILLINOIS ENGINEERING EXPERIMENT STATION

Bulletin Series No. 417

LEAST-WEIGHT PROPORTIONS OF BRIDGE TRUSSES

J. L. Waling

UNIVERSITY OF ILLINOIS BULLETIN

Price: Fifty Cents

UNIVERSITY OF ILLINOIS BULLETIN

Volume 51, Number 13; September, 1953. Published seven times each month by the University of Illinois. Entered as second-class matter December 11, 1912, at the post office at Urbana, Illinois, under the Act of August 24, 1912. Office of Publication, 358 Administration Building, Urbana, Illinois.

UNIVERSITY OF ILLINOIS ENGINEERING EXPERIMENT STATION

Bulletin Series No. 417

LEAST-WEIGHT PROPORTIONS OF BRIDGE TRUSSES

JOSEPH L. WALING

*former Graduate Student
in Civil Engineering*

Published by the University of Illinois, Urbana

ABSTRACT

The determination of least-weight proportions of bridge trusses lends itself to mathematical investigation. The procedure here presented consists of the adaptation of the theory of maxima and minima to solving for the proportions of a truss outline, such that the volume of metal in the truss is a minimum. Explanations are given of the assumptions and approximations upon which the calculations are based and concerning the resolution of complications interspersed by modern design specifications.

Calculations were performed to determine least-weight proportions and theoretical least weights of simple span, through-type, double-track, open-timber-deck railway bridges having sixty-eight different combinations of truss type, panel length, span length, and live load. In general, the results of these calculations show that weight savings can be accomplished by designing these trusses somewhat deeper than is normally done by present-day designers.

This page is intentionally blank.

CONTENTS

I. INTRODUCTION	7
1. Nature of Investigation	7
2. Historical Review	8
3. Purpose and Scope of the Investigation	11
4. Acknowledgments	12
5. Notation	12
II. THE CALCULATIONS	14
6. General	14
7. Basic Assumptions and Approximations	15
8. Assumptions Made for Present Investigation	17
9. Temporary Assumptions	19
10. Solution of Simultaneous Derivative Equations	20
11. Verification of Validity of Temporary Assumptions	21
12. Special Cases — Conflicts	22
13. Closing Remarks Concerning Calculations	26
III. RESULTS	28
14. General	28
15. Least-Weight Depths of Parallel-Chord Trusses	28
16. Least-Weight Depths of Curved-Chord Trusses	32
17. Least Weights of Trusses	35
18. Practical Significance of Results	37
IV. CONCLUSIONS	40
V. BIBLIOGRAPHY	42
APPENDIX A: SAMPLE DERIVATIONS OF GENERAL VOLUME FORMULAS	43
APPENDIX B: GENERAL VOLUME FORMULAS AND DERIVATIVE EQUATIONS FOR WARREN TEN-PANEL CURVED-CHORD TRUSSES	48

FIGURES

1. Influence Line for Stress in U_3-L_3 of P-CC-8	17
2. Weight of Floor vs. Panel Length for Double-Track, Open-Timber-Deck Railway Bridges	18
3. Convergence Diagram	21
4. Simple Conflict	23
5. Trial Solution for Least Volume and Least-Weight Depths When Conflict Exists	24
6. Three-Way Conflict	25
7. Ratios of Least-Weight Depth to Span for Parallel-Chord Trusses	30
8. Variation in d/L Ratio with Number of Panels for Pratt Parallel-Chord Trusses	31
9. Variation in d/L Ratio with Number of Panels for Warren Parallel-Chord Trusses	31
10. Ratios of Least-Weight Depths to Span for Pratt Curved-Chord Trusses	32
11. Variation in d_1/L Ratio with Number of Panels for Pratt Curved-Chord Trusses	33
12. Ratios of Least-Weight Depths to Span for Warren Curved-Chord Trusses	34
13. Variation in d_1/L Ratio with Number of Panels for Warren Curved-Chord Trusses	34
14. Variation of Ratio w_{tb}/w_{etb} with Span for Parallel-Chord Trusses	36
15. Variation of Ratio w_{tb}/w_{etb} with Span for Curved-Chord Trusses	36
16. Influence of Number of Panels on Least Dead Load	37
17. Weight-Depth Curve	39

TABLES

1. Scope of Calculations	11
2. Average Values of C and T	19
3. Least-Weight Characteristics of Six-Panel Bridge Trusses	28
4. Least-Weight Characteristics of Eight-Panel Bridge Trusses	29
5. Least-Weight Characteristics of Ten-Panel Bridge Trusses	29

I. INTRODUCTION

1. Nature of the Investigation

Since the earliest time of scientific design of bridges, engineers have considered the problem of economy in bridge building. While this branch of engineering is an old one and much work has been done in solving its economic problems, still more remains to be done.

Studies on economic bridge design and construction involve many variables or factors. Most important among these are the costs of materials, fabrication, transportation to the site, erection, and maintenance. All of these are influenced by the choice of materials, the choice of general arrangement of the whole structure, the over-all proportioning of the separate structural frameworks, the proportioning of details, the additional provisions for erection and maintenance, and any additional provisions for a pleasing and safe appearance.

Many economic factors in bridge design do not lend themselves to mathematical solutions, and one cannot solve for all of the variables by the use of equations. This affords opportunity for the exercise of engineering judgment—sometimes called common sense—the possession and use of which distinguish the great engineer from the average. On the other hand, solutions of those portions of the economic problem which are adaptable to mathematical analysis can be a valuable aid in the judicious choice of materials and proportions for bridge structures.

One important economic factor in bridge construction which lends itself to mathematical investigation is the quantity of material used in the structure. This is influenced greatly by the general proportions chosen for the main structural framework. In the case of truss bridges, the over-all dimensions of the main trusses, such as the depth of truss and the number of panels for a given span and loading, have a considerable bearing upon the material in the superstructure. The determination of least-weight proportions of bridge trusses designed to satisfy modern specifications is therefore a problem of continuing interest and importance to design engineers. This one portion of the entire problem of economy is the subject of the present investigation.

2. Historical Review

As early as 1847, Squire Whipple^{(1)*} considered the problem of economy of material in bridges. He used mathematical differentiation to determine the inclination of individual members of trusses, such as tension diagonals and compression diagonals, which would require the least material for those particular members when the truss was subjected to a single concentrated load. By deduction from these results, he reasoned that the height of a parallel-chord truss should be about one-sixth of its length. Obviously his results should not be assumed as exactly compatible with modern materials and design specifications.

In 1871, T. Willis Pratt⁽²⁾ discussed the economy of material to be expected in trusses "where the oblique braces are in tension, and the vertical supports [members] in compression." This kind of parallel-chord truss is now known as the Pratt truss.

In 1877, Emil Adler⁽³⁾ stated a criterion for least-weight depth of wrought iron parallel-chord trusses as follows: "For the most economical depth, the material in the two chords together must be equal to the material in the vertical members, plus the material in the inclined members, the latter multiplied by $\cos(180^\circ - 2\alpha)$, where α is the angle the inclined members make with the horizontal chords." This same criterion was demonstrated ten years later in an apparently independent work by John Lundie and was discussed by Scherzer⁽⁷⁾ who stated the assumptions upon which Lundie's demonstration was based and cited its shortcomings; i.e., mainly that, (1) the assumptions do not take into consideration any strut formula for compression members, and (2) the economical depth established for a single panel of a Pratt truss cannot be applied to a whole truss. The use of the Adler (Lundie) rule, if it were valid, would make necessary a trial-and-error solution for the least-weight depth in any practical case.

Charles E. Emery⁽⁴⁾ in 1877 also outlined a trial-and-error procedure for determining least-weight proportions of iron parallel-chord trusses. At that early date he concluded that: "the height [depth] of all forms of bridge, except the continuous girder, may be materially increased — as compared with ordinary practice — with a saving of material, even when proper provision is made to secure the stability of longer struts and counteract the increased effects due to wind pressure. In fact the ordinary heights of some forms of girder may be nearly or quite doubled without loss."

In 1878, William E. Searles⁽⁵⁾ used Emery's results to make a "great saving of about one hundred dollars" by increasing the depth of a wooden parallel-chord truss from 8 ft to 16 ft during its reconstruction.

* Parenthesized superscripts refer to correspondingly numbered entries in the Bibliography.

A. J. Dubois⁽⁶⁾ in 1887 presented some rational but complicated formulas for weights of material in bridges and for economic depths of iron parallel-chord trusses.

In 1895, J. A. L. Waddell⁽⁸⁾ censured writers on the subject of economy in superstructure design, "who invariably attack the problem by means of complicated mathematical investigations, not recognizing the fact that the questions they endeavor to solve are altogether too intricate to be solved by mathematics." He listed the common incorrect assumptions made in such investigations and classified the results as "simply a waste of good mental energy." He then made the assumption that the weight of the chord members in a parallel-chord truss varies inversely as the depth, and the weight of the web members varies directly as the depth of the truss. Hence he proved mathematically that "the greatest economy of material will exist when the weight of the chords is equal to the weight of the web." This criterion, like that of Adler (and Lundie), necessitates a trial-and-error design procedure to determine an approximation to the least-weight depth of a parallel-chord truss.

In 1914, J. Melan⁽⁹⁾ performed calculations to determine the most economical depth and number of panels for two types of parallel-chord trusses, namely, Pratt trusses and Warren trusses without verticals. He formulated mathematical expressions for the volumes of material in the trusses, making use of empirical "Konstruktionkoeffizienten" to account for variations due to column action as well as the various construction details such as gusset plates, lattice bars, batten plates, and rivets. These construction coefficients were assumed to be constant for each group of members, i.e., independent of the length of the individual members. By plotting curves of variation of weights with truss depth and number of panels, he determined the least-weight proportions for the two types of trusses investigated. He stressed the economy to be gained by decreasing the number of panels and increasing the depth of truss.

Maximilian H. Angst⁽¹⁰⁾ in 1915 investigated least-weight proportions for parallel-chord bridge trusses with various web systems, using about the same method of investigation as that of Melan. He stated that the most favorable truss depths of "Halbparabelträgern" are about fifteen percent greater than those for parallel-chord trusses; however, he gave no details on the determination of the depths of the half-parabolic trusses. He summarized his results in part as follows: "This investigation shows distinctly that large panel lengths and high truss depths are indicated for these superstructures. Our [European] bridge constructions are in this respect still much too conservative."

In 1931, Adolf Voigt⁽¹¹⁾ also made a theoretical investigation of the most favorable height of steel railway bridge trusses. He depended upon estimated values of the "Bauziffer" — construction coefficient — as did Melan and Angst. However, he used the expedient of differentiating the expression for total volume of metal with respect to the depth of truss to determine the least-weight depth. He investigated parallel-chord trusses having Warren-type web systems with verticals and subdivided panels. He summarized the results of this investigation by stating that the angle of inclination of the web diagonals from the horizontal, corresponding to the most favorable depth of truss, is approximately $\arctan 2.1$.

In 1950, Otfried Erdmann⁽¹²⁾ investigated the influence of high strength steel on economy in steel bridge construction. In his theoretical formulas for weights of trusses, he introduced five different factors or coefficients which taken together correspond to the construction coefficients proposed by previous European investigators. The five coefficients were used to correct his "basic weights" for additions due to:

- (1) Reduction of net areas by rivet holes in tension members
- (2) Allowance for buckling of compression members
- (3) Allowance for reversal of stress
- (4) Overdesign of members caused by the limited variety of sizes of rolled shapes available
- (5) Weight of gusset plates, splices, lattice bars, batten plates, rivet heads, and tolerances of rolling

Only parallel-chord trusses were included in the research by Erdmann and no attempt was made to determine most favorable truss depths.

In 1948, Melvin W. Jackson⁽¹³⁾ completed an investigation of the least-weight depths of Warren parallel-chord railway bridge trusses. He made preliminary designs for some two hundred bridges in accordance with AREA⁽¹⁴⁾ specifications and determined the truss depths corresponding to minimum weight by plotting of weight curves. He formulated an empirical expression for the truss depth as follows:

$$d = 0.21 p \sqrt{n} \sqrt[3]{W} \quad (1)$$

in which

d is the least-weight depth

p is the panel length

n is the number of panels

W is the load in kips per ft of bridge which under the conditions set will produce the maximum chord stress

A research similar to that of Jackson is being carried out by Aly S. Shoukry, a graduate student at the University of Illinois, to determine least-weight depths of Warren polygonal-chord trusses.

The foregoing historical summary of research completed on least-weight proportions of trusses clearly indicates trends in thinking by engineers of different nationalities. Several early Americans attempted theoretical investigations of increasing complexity and accuracy until 1895 when Waddell expressed his opinion of such mathematical calculations. Thereafter, the mathematical approach to the problem was employed primarily by Europeans—mostly by German engineers. Americans came to rely upon results of practical experience or upon a great number of actual designs to study the question and to arrive at empirical formulas for most favorable proportions of trusses.

Nearly all of the investigations apply to parallel-chord trusses; only the theoretical results mentioned by Angst⁽¹⁰⁾ for the "Halbparabel-trägern" and the practical investigation in progress by Shoukry pertain to curved- or polygonal-chord trusses. Much work therefore remains to be done for the latter type of bridge truss.

3. Purpose and Scope of the Investigation

The objects of this research were twofold: (1) to develop a sound mathematical procedure, founded upon modern design specifications, for the determination of least-weight proportions of bridge trusses; and (2) to apply the mathematical procedure to investigate least-weight proportions for a series of railway bridge trusses, including both parallel- and polygonal-chord* trusses.

Table 1
Scope of Calculations

	Number of Panels			Span (ft)
	6	8	10	
Length of Panels (ft)	28			168
	32	24*		192
	36*	27	21.6	216
	40	30	24*	240
		35	28	280
		37.5*	30	300

* E 75 live loading used.

This investigation was made for through-type double-track railway bridges having open timber decks. It includes calculations of least-weight depths and of the least weight for each of the panel-span combinations listed in Table 1. Each combination was applied for both Warren and Pratt web systems, for parallel- and curved-chord trusses, using Cooper E-60 live loading for each, and E-75 live loading for those indicated by an asterisk. In all, least weights and least-weight depths were calculated for 68 truss arrangements.

* Hereafter to facilitate identification of truss forms by notation, polygonal-chord trusses are called curved-chord trusses.

4. Acknowledgments

This bulletin is an outgrowth of a thesis presented in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering at the University of Illinois. The original work and the transition from thesis to bulletin were done under the helpful direction of Thomas C. Shedd, Professor of Structural Engineering in the Department of Civil Engineering.

Grateful acknowledgment is made of a sabbatical leave granted by Purdue University to the writer in order that he might complete the work for the doctorate at the University of Illinois.

5. Notation

The following notation is used throughout this bulletin:

- C (kips/in.²) = allowable compression stress on the gross area of a truss member
- d (ft) = depth of parallel-chord truss from axis of compression chord to centerline of tension chord
- d_1 (ft) = depth of polygonal-chord truss at the first interior panel point
- d_2 (ft) = depth of polygonal-chord truss at the center of the span
- h (in.) = over-all depth of cross-section of a truss member
- k = the ratio d_1/d_2
- L (ft) = length of span of truss
- l (ft) = length of a member of a truss, panel point to panel point
- n = number of panels in truss span
- p (ft) = length of each panel in truss
- r (in.) = least radius of gyration of cross-section of a truss member
- S_{\max} (kips) = maximum stress
- S_{\min} (kips) = minimum stress
- $S_{D,\max}$ (kips) = maximum design stress
- $S_{D,\min}$ (kips) = minimum design stress
- T (kips/in.²) = allowable tensile stress on the *gross* area of a truss member
- V (in.²ft) = volume of metal in a truss member
- $V_{(c)}$ (in.²ft) = volume of metal in a truss member, governed by compressive stress
- $V_{(t)}$ (in.²ft) = volume of metal in a truss member, governed by tensile stress
- $V_{(l/r)}$ (in.²ft) = volume of metal in a truss member, governed by slenderness ratio
- $V_{1/2}$ (in.²ft) = total volume of metal in one-half of the length of the truss

w_{tb} (kips/ft) = weight of one truss and one-half of bracing, considered uniformly distributed over the length of the truss span

w_{etb} (kips/ft) = weight on one truss exclusive of truss and bracing, considered uniformly distributed over the length of truss span; this includes floor, live, and impact loads

w_1 (kips/ft) = distributed dead load carried by one truss

w_2 (kips/ft) = sum of live load and impact load uniformly distributed over any portion of one truss to produce maximum stress in a member

The following abbreviations hardly need explanation: CC, curved-chord, to designate polygonal-chord type of truss; DL (kips/ft), dead load; E-60, designation of magnitude of Cooper's live load; E-75, proportional to E-60; LL (kips/ft), live load; M (ft kips), bending moment; P, designation for Pratt-type web system; PC, parallel-chord, to designate type of chord system; W, designation for Warren-type web system. P-PC-8 @ 30 = 240, E-60 is a type of designation composed from the above to describe a type of truss, number and length of panels, total length of span and design live load; this example indicates a Pratt parallel-chord truss of 8 panels at 30 ft each, giving a span of 240 ft, designed for Cooper's E-60 live load.

II. THE CALCULATIONS

6. General

The procedure developed and used in accomplishing the objectives of this research consists of the adaptation of the mathematical theory of maxima and minima to the determination of the depth, or the depths in the case of curved-chord trusses, at which the volume of metal in the truss becomes a minimum. This procedure is complicated only by the necessity of conforming to all of the requirements of modern design specifications.⁽¹⁴⁾

The volume V of the geometric length of each member of the truss in one-half span length (in cases of symmetrical trusses) is formed algebraically in terms of w_1 , w_2 , p , T , C , d_1 , and d_2 .

$$V_{\frac{1}{2}} = \Sigma V \quad (2)$$

where the summation is a function containing the constants w_1 , w_2 , p , T , and C and the variables d_1 and d_2 .

The necessary conditions

$$\frac{\partial \Sigma V}{\partial d_1} = 0 \quad \text{and} \quad \frac{\partial \Sigma V}{\partial d_2} = 0 \quad (3)$$

for locating a maximum or a minimum volume can be written as

$$\Sigma \frac{\partial V}{\partial d_1} = 0 \quad \text{and} \quad \Sigma \frac{\partial V}{\partial d_2} = 0 \quad (3a)$$

The application of these two conditions furnishes two algebraic equations which for simplicity are herein called derivative equations. Appropriate values for all the terms except d_1 and d_2 are substituted into these equations. Except in special cases which will be discussed in detail, simultaneous solution of the two equations then determines values of d_1 and d_2 for which the volume of metal in the truss is a minimum. As Sherwood and Taylor⁽¹⁵⁾ and other authors on calculus have explained, when practical experience indicates that a minimum volume exists in the explored region, the condition

$$\left(\frac{\partial^2 \Sigma V}{\partial d_1 \partial d_2} \right)^2 - \frac{\partial^2 \Sigma V}{\partial d_1^2} \cdot \frac{\partial^2 \Sigma V}{\partial d_2^2} < 0 \quad (4)$$

which is sufficient to assure the existence of a maximum or minimum need not be studied. That the solution of the two equations locates a minimum

and not a maximum volume can be verified, if desired, by inspection of the second partial derivatives which satisfy the conditions

$$\sum \frac{\partial^2 V}{\partial d_1^2} > 0 \quad \text{and} \quad \sum \frac{\partial^2 V}{\partial d_2^2} > 0 \quad (5)$$

The least weight of truss is then determined by substituting the derived values of d_1 and d_2 into the terms contained in the expression for volume of one-half truss and converting the sum of the volumes into a corresponding distributed weight.

In the case of parallel-chord trusses, only one depth must be determined; consequently only one derivative equation is required. Otherwise the procedure is the same.

While the foregoing basic procedure is straightforward, there are several assumptions, procedural details, and special cases which warrant further explanation and discussion. These are considered subsequently and separately.

7. Basic Assumptions and Approximations

In order to calculate the least-weight proportions of a structure it is necessary to consider certain features of the structure as already set. In the present case, it is logical to assume that the truss span length, number of panels, and general arrangement of truss members are known.

For a curved-chord truss it is expedient to assume the general shape of the upper chord in order to limit the number of unknown depths to two. In the derivation of the general volume formulas for the curved-chord trusses investigated, the top chord panel points are assumed to lie on a parabolic curve which passes through the first interior and center top chord panel points. This assumption conforms to common design practice. Analysis of parallel-chord trusses involves no such assumption.

The dead load on the truss, consisting of floor weight, truss weight, and bracing weight, is assumed as usual in preliminary design to be uniformly distributed over the full length of span.

Uniform loads equivalent to the E-loadings are used in this investigation; furthermore it is assumed that the magnitude of the distributed live load is the same for all the members of a given truss. Likewise the impact factor specified by AREA⁽¹⁴⁾ is assumed to be the same for hangers as for all other members. Both the live load and the impact factor are determined for each structure in accordance with the total length of span; i.e., the same value is used for the web members and the chord members as would normally be used for the chord members. This approximation is not absolutely necessary, but it is advantageous in

reducing the amount of work while introducing comparatively small error for web members only. Moreover, this error is at least partially compensated by another simplifying approximation which is made in formulating the expression for the volume of each web member.

In deriving the algebraic expressions for the volume of metal in web members, only full panel lengths of live load are used. That is, when the influence line indicates partial loading on a panel for maximum stress of a given sign, the panel is loaded completely and the influence line treated as though extending to the next panel point. An example of this approximation is shown in Fig. 1. In determining the maximum positive live load stress in member U_3-L_3 , the four full panels on the left are loaded and the line abc is taken as an approximation to the correct influence line in that region. Since the position of zero influence depends upon the variables d_1 and d_2 , it is obvious that this approximation affords considerable simplification while introducing no great error. The approximation is always on the conservative side and tends to compensate for the error introduced in the expressions for volume of the web members by assuming the same equivalent uniform load for web members as for chord members.

An idea of the degree of compensation and the final effect of the two preceding approximations is obtained by a study of the truss W-CC-8 @ 37.5 = 300, E-60. After initial solution of the simultaneous derivative equations, the volumes required in the web members for live and impact load are found to be 3.8 to 4.8 percent greater than those established by the two approximations. The live load volume of only one-fifth of the members of the truss are involved in these approximations; and the least-weight portal and midspan depths are changed from 50.3 to 50.4 ft and from 73.3 to 72.9 ft respectively — a maximum change of only about one-half of one percent.

Wind loads are not considered since the AREA specification allows a 25 percent increase in basic intensity of stress when wind stresses are added to those caused by live load, impact, and dead load. Because of this allowance, wind loads require little if any additional metal in the trusses of double track bridges of the lengths considered here.

T and C are symbols introduced in the expressions for volumes of members governed by stress. It is assumed in the derivation of the general equations that values of T and C can be established approximately for each member in any given truss. The determination of specific values to be used for these allowable stresses in the present investigation is discussed in the next section.

To form the algebraic expression for the volume of a web member which is possibly governed entirely by the AREA requirements for slenderness ratio, it is necessary to assume the general form of the cross-section. For example, in this research it is assumed that all such members consist of two web plates and four angles, riveted and latticed together as indicated in the sample volume formula derivation which is included in Appendix A.

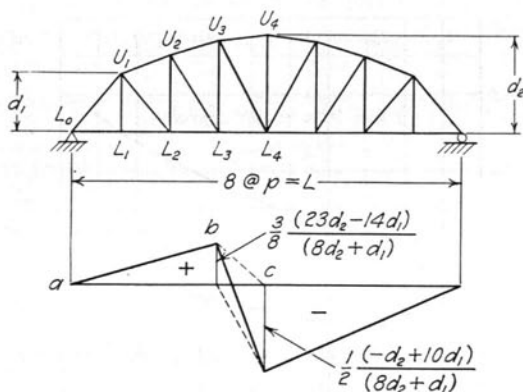


Fig. 1. Influence Line for Stress in U_3-L_3 of P-CC-8

Alternate expressions for volume are necessary for certain web members of each type of truss — most often in curved-chord trusses. In these the resulting magnitude of the ratio of d_2 to d_1 determines not only whether or not the member is subjected to reversal of stress, but also whether its volume is governed by tension as the maximum design stress, compression as the minimum but governing stress, or compression as the maximum design stress. An alternate expression for volume governed by slenderness ratio requirements corresponding to the governing stress is also needed for such members. Few alternate terms are required for parallel-chord trusses since the character of the governing stress for most of the members can be determined by inspection.

Sample general formulas for volume of metal and the corresponding derivative equations for Warren 10-panel curved-chord trusses are tabulated in Appendix B. Sample derivations are given in Appendix A.

8. Assumptions Made for Present Investigation

The work previously done by M. W. Jackson⁽¹³⁾ furnishes some excellent data. The results of his designs of floor systems for double-track open

timber deck for panel lengths of 24, 30, and 36 ft provide floor weights which are plotted on Fig. 2. The curves corresponding to E-60 and E-75 live loads are extrapolated to cover the range of panel lengths from 21 to 40 ft. These data are used throughout this study.

In preparation for this investigation of least-weight truss depths a study was made to determine satisfactory values for C and T for the

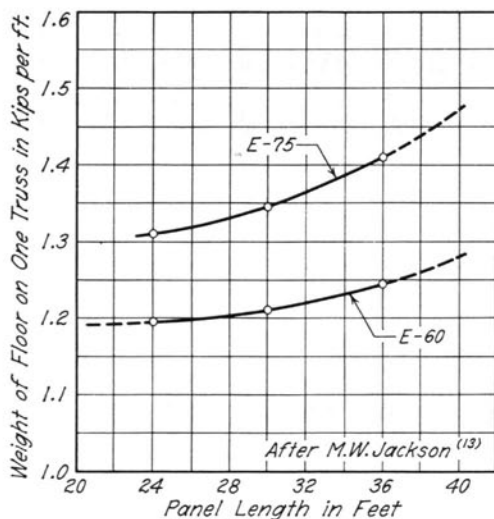


Fig. 2. Weight of Floor vs. Panel Length for Double-Track, Open-Timber-Deck Railway Bridges

various members of the trusses. Magnitudes of C and T resulting from the study of Jackson's design tabulations for Warren parallel-chord trusses are the basis for the average values for the groups of members (except compression verticals) shown in Table 2. C for compression verticals of Pratt trusses is arbitrarily taken as 13.0 to 13.5 kips per sq in. unless the volume required by buckling indicates that a lower value should be used in a particular case. Otherwise Table 2 gives the basis for assigning magnitudes of constants C and T throughout the calculations.

To arrive at an estimate of the total distributed weight of truss, 37.5 percent of the weight of bare members is added to the latter as allowance for weight of details such as gusset plates, splices, lattice bars, batten plates, rivet heads and tolerances of rolling. That this is an acceptable figure is indicated in work by Professor Shedd.⁽¹⁶⁾

The weight of lateral bracing carried by each truss is taken as 12.5 percent of the total weight of truss (including bare members plus details). Professor Shedd shows an example of a bridge of 300-ft span, the weight of bracing for which is 11.3 percent of the truss weight. Kunz,⁽¹⁷⁾ in a tabulation of detailed weights of bridges designed according to the American Bridge Company's specifications of 1900 for E-50 loading, shows examples in which the weight of bracing is 12.8 percent and 11.0 percent of the truss weight for spans of 150 ft and 200 ft respectively. Waddell⁽¹⁸⁾ shows corresponding percentages for Pratt curved-chord

Table 2
Average Values of C and T

<i>Types of Members</i>	<i>C (kips/sq in.)</i>	<i>T (kips/sq in.)</i>
Chords, Tension		14.6
Chords, Compression	14.6	
End Posts	14.0	
Vertical Hangers		14.2
Web Diagonals, Tension		14.6
Web Diagonals, Compression	12.5-13.0	
Web Verticals,* Compression	13.0-13.5	
Web Verticals,* Tension		14.6

* Pertain only to members of Pratt trusses

trusses ranging from 10.4 percent for spans of 300 ft to 16.6 percent for spans of 180 ft; for Pratt parallel-chord trusses he shows the weight of bracing as 14.3 percent and 15.6 percent of truss weight for spans of 200 ft and 180 ft respectively. In view of the practical experience represented by the foregoing examples, the use of 12.5 percent seems justified.

The slenderness ratio for sub-verticals of Warren trusses is limited to 120 in this research and their volume expressions are set up accordingly. The maximum slenderness ratio of all other members is that which is specified by AREA.⁽¹⁴⁾

9. Temporary Assumptions

In designing a bridge truss, one must first estimate the weight of the truss and bracing; likewise preparatory to solving for least-weight depths of a truss, one must estimate or assume the weight of the truss and bracing. For this investigation, the ratio w_{tb}/w_{etb} is employed as a convenient measure of the weight of the truss and bracing. In studying a series of similar trusses of different spans, it is convenient to plot the ratios w_{tb}/w_{etb} versus spans, as calculated, and to extrapolate the resulting curve to estimate the weight of the next truss in the series. The estimate of truss weight is one which can be checked after the most favorable depths are determined; it is therefore considered as a temporary estimate which can be made as accurate as desired by successive calculations.

Where alternate algebraic formulas represent possibilities for governing the volume of a member, the most likely one must be chosen in preparation for the solution of the simultaneous derivative equations. The choice between the volume functions governed by stresses is facilitated by a judicious guess of what value of the ratio d_2/d_1 will result in the least weight of truss. In addition, the volume expression set up to satisfy the slenderness ratio requirement for the member must be considered; only experience can guide in choosing between the latter and the corresponding stress-governed function. However, since this choice is always checked when one solves for the total volume of metal in the truss, a trial-and-error procedure is used to gain the necessary experience.

10. Solution of Simultaneous Derivative Equations

The derivative equations are algebraic equations which contain non-linear functions of the two unknowns d_1 and d_2 in the case of curved-chord trusses and of the one unknown d in the case of parallel-chord trusses. The simultaneous solution of the two equations involving d_1 and d_2 is accomplished by an iterative procedure as follows:

(1) Estimate the value of the ratio d_2/d_1 corresponding to the least weight of truss.

(2) Using the estimated value of the ratio d_2/d_1 , substitute into the number one derivative equation in terms of d_1 and solve that equation for the first approximation of d_1 .

(3) Likewise, substitute into the number two derivative equation in terms of d_2 and solve for the first approximation of d_2 .

(4) The first derived values of d_1 and d_2 give a new value for the ratio d_2/d_1 (unless the first estimate was correct).

(5) Use the new value of the ratio d_2/d_1 (unless experience permits a still better estimate) and repeat steps (2) through (4) to obtain a better approximation to the correct values of d_1 and d_2 .

(6) Convergence to the correct values of d_1 and d_2 is then facilitated by the use of a convergence diagram, a sample of which is shown on Fig. 3. Assumed or trial ratios d_2/d_1 are plotted as abscissas while the corresponding derived ratios are plotted as ordinates. Two assumed and the corresponding two derived values of the ratio d_2/d_1 plot as two points on the graph. A straight line drawn through the two points intersects the 45-deg line represented by the equation d_2/d_1 (derived) = d_2/d_1 (assumed) at a very close approximation to the correct ratio d_2/d_1 . If the original estimate of the ratio d_2/d_1 is not too much in error, the straight line through the two points intersects the equality line at a point which can be taken to represent the correct value of the ratio; otherwise

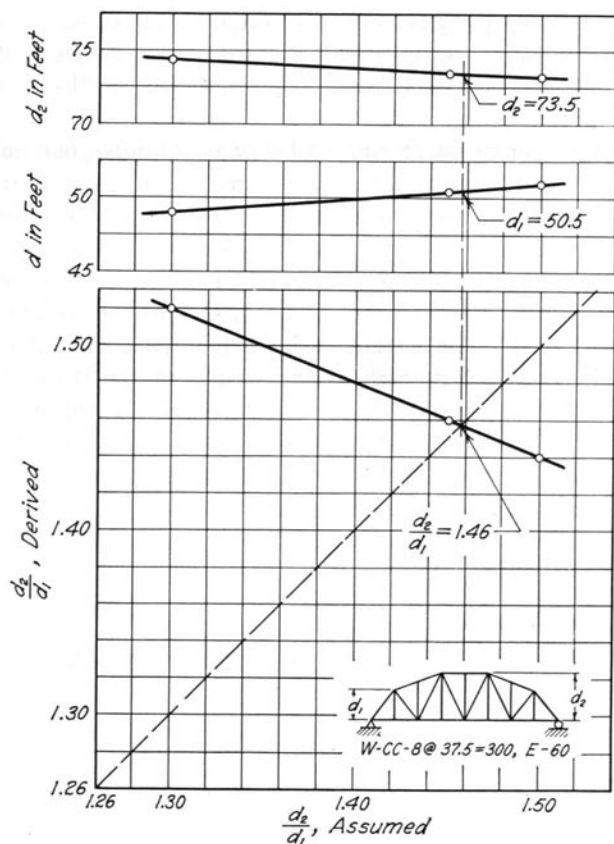


Fig. 3. Convergence Diagram

steps (2) through (4) should be repeated to check the validity of the approximation obtained from this graph. The additional curves shown at the top of Fig. 3 are used to obtain correct values of the depths d_1 and d_2 corresponding to the converged ratio d_2/d_1 .

11. Verification of Validity of Temporary Assumptions

Convergence to the correct values of d_1 and d_2 which result from the temporary assumptions discussed under Section 9 permits one to confirm the validity of those assumptions. In the first place when the depths of truss become known, the initial assumptions concerning reversal of stress and governing stress (or governing volume formula) can be confirmed for each web member. If an incorrect choice of volume formula has been

made, the computed depths usually give a better indication of the correct choice; the calculations are repeated for that new choice until the temporary assumptions are each compatible with the depths d_1 and d_2 resulting therefrom.

When the above condition of compatibility is attained, one can substitute the values of d_1 and d_2 into the volume formulas to obtain the magnitude of the total volume of metal in the bare members of one-half of one truss. The volume of metal is converted to an equivalent distributed weight and 37.5 percent of this weight is added as allowance for the construction details listed in Section 8; this gives the weight of the one truss in kips per ft. The weight of lateral bracing carried by each truss is then taken as 12.5 percent of the weight of the truss, and the combined weight of truss and bracing is obtained. This is compared with the estimated weight of truss and bracing to determine the validity of the initial estimate. In this investigation, if the resulting dead load — weight of floor, truss, and bracing — differs by as much as one percent from that which has been assumed, the calculations are repeated until agreement within one percent is attained.

12. Special Cases — Conflicts

One is not always able to locate a mathematical minimum point in the total volume function, which is compatible with the original choice of volume governing factors. When this difficulty arises, it is usually only one web member of the truss which cannot finally be made to satisfy the compatibility requirement in the normal way already outlined. This situation develops in a manner which seems best described by the term *conflict*. As an example, suppose it has been assumed that tension stress governs the design of a web member, and at depths d_1 and d_2 thus calculated it is found that the minimum stress (compression) would govern the volume of that member. The calculation would be repeated with the assumption now being made that the correct volume expression for the web member is the one governed by the minimum stress; but it then turns out that the newly calculated values of d_1 and d_2 would cause the design of the member in question to be governed by tension. In this situation it is common for all other members to satisfy the compatibility requirements at both sets of depths. The conflict is therefore usually confined to one last member.

A visual representation of a conflict is shown by the three-dimensional graph in Fig. 4. The total volume is plotted on the vertical ordinate from the horizontal plane described by the perpendicular axes d_1 and d_2 . The

volume is then represented by a surface or surfaces lying above the plane of the horizontal axes. Obviously the surface representing the total volume when the volume of the conflict member is assumed to be governed by tension is different from the surface when the member is assumed to be governed by compression. The former is represented in Fig. 4 by the

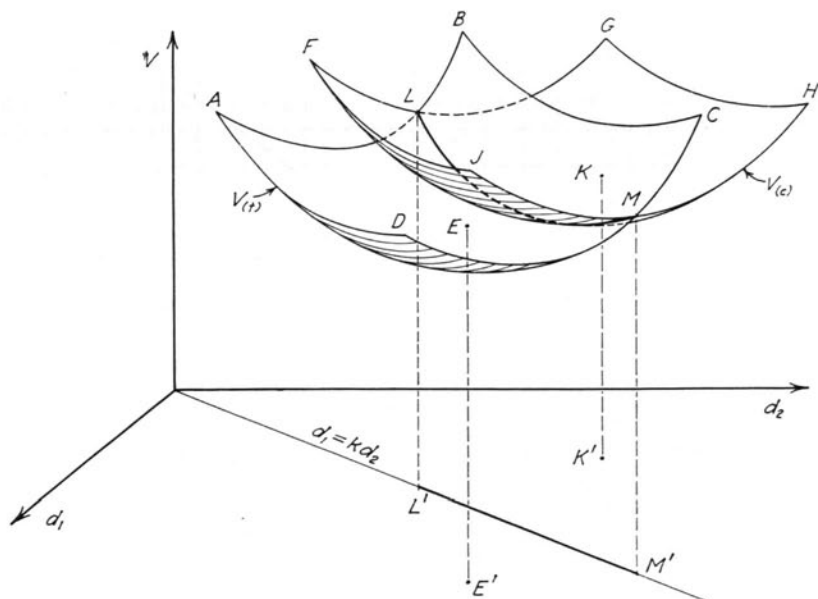


Fig. 4. Simple Conflict

surface $ABCD$ which has a minimum point at E ; the latter is represented by the surface $FGHJ$ which has a minimum point at K . The difference between the two surfaces is fairly small since it consists of the change of volume expression for only the one conflicting member of the truss. The surfaces shown in the figure are exaggerated in the vertical direction for the sake of clarity.

It is seen that at depths corresponding to the minimum point of each of these surfaces, $ABCD$ and $FGHJ$ respectively, the other surface would govern in design. It is also clear that the combined surface $BCJF$, consisting of a portion of each of the two original surfaces which intersect on the valley curve LM , is the true governing surface for the truss. The true value of least volume and the corresponding depths are therefore determined by locating the minimum point on the valley curve LM .

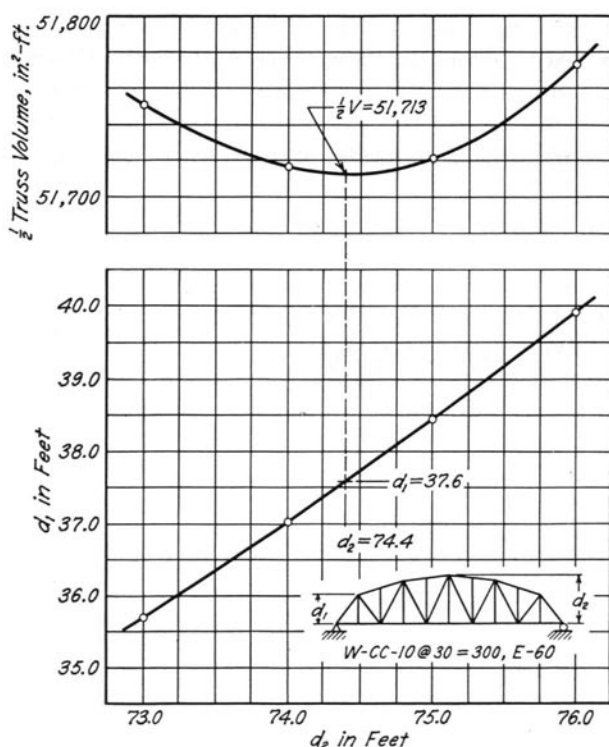


Fig. 5. Trial Solution for Least Volume and Least-Weight Depths When Conflict Exists

Since the so-called valley curve LM is the intersection of the two original surfaces, the relation between d_1 and d_2 on this curve is determined by equating the two conflicting volume expressions for the web member in question. When both of the two conflicting volume expressions are governed by stress and neither by slenderness ratio requirements, the relation between the two depths is linear; i.e., it is in the form

$$d_1 = kd_2 \quad (6)$$

in which k is a constant. The original volume formulas are then transformed into a new set in terms of the constant k and one variable d_2 . Either of the two conflicting terms for the volume of the one web member can be used in the transformed set. Equating to zero the derivative of the sum of new volume formulas with respect to the one variable d_2 furnishes one new derivative equation from which d_2 can be determined. Hence d_1 is also determined by virtue of Eq. 6.

When one of the two conflicting volume expressions for the one web member is governed by stress and the other is governed by slenderness ratio specifications, the relation between d_1 and d_2 resulting from equating the two volume components is neither a constant nor a simple function. The solution for least volume and least-weight depths is then most easily obtained by a semi-graphical calculation, a sample of which is summarized by Fig. 5. A curve relating d_1 and d_2 is plotted as shown at the bottom of the figure. This curve, which is obtained by equating the two conflicting volume terms, is the projection of the so-called valley curve onto the d_1, d_2 plane; it limits the combinations of d_1 and d_2 which can possibly lead to least weight of metal in the truss. Combinations of the depths represented by points on this curve are then substituted into the general volume formula to obtain data for the curve of volumes plotted at the top of the figure. The minimum point of the volume curve is obtained by inspection, and the corresponding least-weight depths are furnished by the lower curve.

Infrequently a case arises wherein a three-way conflict occurs; it is usually encountered in the following manner. The minimum point on the valley curve LM , Fig. 6, is obtained only to find that the volume required by slenderness ratio of the conflict member is slightly greater than that common volume required by the two kinds of stresses. Since the volume

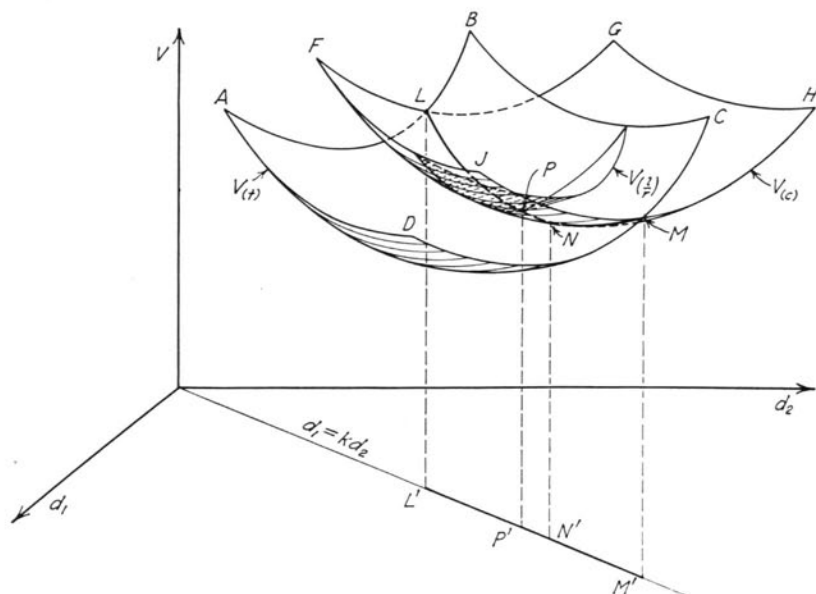


Fig. 6. Three-Way Conflict

required by slenderness ratio increases relatively rapidly with increasing truss depths, it is fairly clear that the valley curve LM ceases to be the governing curve at the point of intersection P of the three conflicting surfaces. The least-weight depths are then determined for this intersection point by solution of the two independent simultaneous equations obtained by equating the three conflicting volume terms.

The foregoing discussion of conflicts pertains primarily to curved-chord trusses; however, the same principles are involved in simpler combinations in the analysis of parallel-chord trusses.

13. Closing Remarks Concerning the Calculations

Re-examination of these calculations for the purpose of comparing them with the theoretical works of Melan, Angst, Voigt, and Erdmann reveals some similarities and some basic differences in the general method of approach. In general, all of these investigations start with the formulation of an algebraic expression for the volume of each member of the truss being studied. The European engineers based the volume expression in each case upon the maximum stress applied to the member and relied upon the use of one or more construction coefficients (multipliers) to account for the additional volume required by all or some of the following factors:

- (1) Reduction of net areas by rivet holes in tension members
- (2) Allowance for buckling of compression members
- (3) Allowance for reversal of stress
- (4) Allowance for overdesign of members caused by the limited variety of sizes of rolled shapes available
- (5) Weight of gusset plates, splices, lattice bars, batten plates, rivet heads, and tolerances of rolling

Erdmann used a separate construction coefficient for each of these five factors; his five coefficients afforded more comprehensive coverage of the variables involved than the construction coefficients used by the previous investigators.

For the present calculations, the statistical review of the results of Jackson's designs to establish representative values of the allowable tensile stresses T serves the same purpose as the choice of the first of Erdmann's coefficients. Likewise the establishment of the allowable stresses C for the various groups of members whose volumes are governed by compressive stresses obviates the use of Erdmann's second coefficient for these members. To meet the requirements of the AREA specification which limits the maximum slenderness ratio of the various members,

alternate algebraic expressions which are independent of the stresses carried by the members are used where applicable in the present study. This procedure replaces the use of Erdmann's second coefficient for compression and/or tension members carrying little or no stress; it should produce a more representative influence upon the results than use of his second coefficient alone.

Since the AREA specifications pertaining to reversal of stress are followed in setting up the volume terms for the present study, no factor similar to Erdmann's third coefficient is required. His fourth and fifth coefficients correspond to the constant 37.5 percent addition to the bare weight of all the members to allow for the weight of the various details.

The present and previous investigations are similar and dissimilar in other ways made apparent by the historical review in Section 2.

III. RESULTS

14. General

In this study, calculations furnish the least-weight characteristics of double-track railway bridge trusses having the 68 different combinations of truss type, panel length, span length, and live load listed in Table 1. The results of these calculations are summarized in Tables 3, 4, and 5 and provide the data for the curves which follow. In general, the calculated data points are not plotted on the curves which are drawn for the various trusses except when points are used to indicate the isolated data obtained for the E-75 loading.

Table 3
Least-Weight Characteristics of Six-Panel Bridge Trusses

Truss Type	p	L	Live Load	d_1/L	d_2/L	w_{1b}	w_1	w_2	w_{1b}/w_{elb}
	(ft)	(ft)				(k/ft)	(k/ft)	(k/ft)	
P-CC-6	28	168	E-60	0.252	0.365	0.920	2.123	9.183	0.089
	32	192	E-60	0.247	0.346	1.022	2.242	8.770	0.102
	36	216	E-60	0.241	0.318	1.126	2.371	8.414	0.117
	36	216	E-75	0.251	0.358	1.383	2.793	10.518	0.116
	40	240	E-60	0.235	0.298	1.242	2.522	8.128	0.132
W-CC-6	28	168	E-60	0.242	0.346	0.930	2.133	9.183	0.090
	32	192	E-60	0.235	0.334	1.040	2.260	8.770	0.104
	36	216	E-60	0.213	0.276	1.199	2.444	8.414	0.124
	36	216	E-75	0.218	0.287	1.440	2.850	10.518	0.121
	40	240	E-60	0.211	0.269	1.341	2.621	8.128	0.143
P-PC-6	28	168	E-60		0.280	1.010	2.213	9.183	0.097
	32	192	E-60		0.278	1.127	2.347	8.770	0.113
	36	216	E-60		0.276	1.247	2.492	8.414	0.129
	36	216	E-75		0.279	1.533	2.943	10.518	0.128
	40	240	E-60		0.270	1.369	2.649	8.128	0.146
W-PC-6	28	168	E-60		0.275	1.006	2.209	9.183	0.097
	32	192	E-60		0.271	1.125	2.345	8.770	0.113
	36	216	E-60		0.269	1.246	2.491	8.414	0.129
	36	216	E-75		0.273	1.529	2.939	10.518	0.128
	40	240	E-60		0.266	1.377	2.657	8.128	0.146

15. Least-Weight Depths of Parallel-Chord Trusses

The influence of number of panels and span length on the ratios of least-weight depths to spans for parallel-chord trusses having Pratt and Warren web systems is shown graphically on Fig. 7. Since the d/L ratios for trusses with a given number of panels are very nearly constant for all of the span lengths investigated, it is apparent that the least-weight d/L ratios are practically independent of span length; i.e., the function which is to be derived to approximate the least-weight depth of a parallel-chord truss must contain the first power of span length L .

The influence of magnitude of live load on the least-weight d/L ratios is seen to be almost negligible, since the isolated data points corresponding to E-75 loading lie only very slightly above the curves which represent the results for E-60 loading. The number of panels in a Pratt or Warren parallel-chord truss has the greatest influence upon its least-weight d/L ratio; this is indicated by the vertical spacing of the separate curves of each family, corresponding to the number of panels in the truss.

Table 4
Least-Weight Characteristics of Eight-Panel Bridge Trusses

<i>Truss Type</i>	<i>p</i>	<i>L</i>	<i>Live Load</i>	<i>d</i> ₁ / <i>L</i>	<i>d</i> ₂ / <i>L</i>	<i>w</i> _{th}	<i>w</i> ₁	<i>w</i> ₂	<i>w</i> _{th} / <i>w</i> _{eth}
	(ft)	(ft)				(k/ft)	(k/ft)	(k/ft)	
P-CC-8	24	192	E-75	0.166	0.316	1.357	2.667	10.962	0.111
	24	192	E-60	0.173	0.290	1.111	2.306	8.770	0.112
	27	216	E-60	0.172	0.285	1.234	2.434	8.414	0.128
	30	240	E-60	0.170	0.280	1.361	2.571	8.128	0.146
	35	280	E-60	0.159	0.265	1.587	2.824	7.755	0.176
	37.5	300	E-60	0.155	0.259	1.715	2.972	7.628	0.193
	37.5	300	E-75	0.168	0.281	2.066	3.497	9.534	0.188
W-CC-8	24	192	E-75	0.177	0.276	1.333	2.643	10.962	0.109
	24	192	E-60	0.176	0.267	1.105	2.300	8.770	0.111
	27	216	E-60	0.173	0.260	1.235	2.435	8.414	0.128
	30	240	E-60	0.171	0.254	1.368	2.578	8.128	0.146
	35	280	E-60	0.169	0.246	1.605	2.842	7.755	0.178
	37.5	300	E-60	0.168	0.244	1.739	2.996	7.628	0.196
	37.5	300	E-75	0.171	0.253	2.093	3.524	9.534	0.191
P-PC-8	24	192	E-75	0.235	0.618	2.928	10.962	0.132	
	24	192	E-60	0.234	1.317	2.512	8.770	0.132	
	27	216	E-60	0.234	1.448	2.648	8.414	0.151	
	30	240	E-60	0.233	1.590	2.800	8.128	0.170	
	35	280	E-60	0.232	1.834	3.071	7.755	0.204	
	37.5	300	E-60	0.232	1.971	3.228	7.628	0.222	
	37.5	300	E-75	0.234	2.420	3.851	9.534	0.221	
W-PC-8	24	192	E-75	0.242	1.511	2.821	10.962	0.123	
	24	192	E-60	0.238	1.236	2.431	8.770	0.124	
	27	216	E-60	0.236	1.366	2.566	8.414	0.142	
	30	240	E-60	0.234	1.504	2.714	8.128	0.161	
	35	280	E-60	0.230	1.744	2.981	7.755	0.194	
	37.5	300	E-60	0.229	1.878	3.135	7.628	0.211	
	37.5	300	E-75	0.233	2.297	3.728	9.534	0.209	

Table 5
Least-Weight Characteristics of Ten-Panel Bridge Trusses

T_{russ} T_{type}	p	L	$Live$ $Load$	d_1/L	d_2/L	w_0	w_1	w_2	w_0/w_1 w_1/w_2
	(ft)	(ft)				(k/ft)	(k/ft)	(k/ft)	
P-CC-10	21.6	216	E-60	0.134	0.278	1.311	2.505	8.414	0.131
	24	240	E-60	0.132	0.275	1.442	2.637	8.128	0.155
	24	240	E-75	0.136	0.284	1.756	3.066	10.160	0.153
	28	280	E-60	0.130	0.270	1.671	2.874	7.755	0.187
	30	300	E-60	0.129	0.268	1.799	3.009	7.628	0.204
W-CC-10	21.6	216	E-60	0.126	0.270	1.292	2.486	8.414	0.134
	24	240	E-60	0.125	0.265	1.431	2.626	8.128	0.153
	24	240	E-75	0.126	0.275	1.725	3.035	10.160	0.150
	28	280	E-60	0.123	0.256	1.683	2.886	7.755	0.188
	30	300	E-60	0.125	0.248	1.818	3.028	7.628	0.206
P-PC-10	21.6	216	E-60	0.204		1.648	2.842	8.414	0.172
	24	240	E-60	0.204		1.804	2.999	8.128	0.194
	24	240	E-75	0.205		2.222	3.532	10.160	0.194
	28	280	E-60	0.204		2.078	3.281	7.755	0.232
	30	300	E-60	0.204		2.231	3.441	7.628	0.252
W-PC-10	21.6	216	E-60	0.213		1.490	2.684	8.414	0.155
	24	240	E-60	0.211		1.640	2.835	8.128	0.176
	24	240	E-75	0.214		2.007	3.317	10.160	0.175
	28	280	E-60	0.209		1.901	3.104	7.755	0.212
	30	300	E-60	0.207		2.045	3.255	7.628	0.231

Figure 8 shows the variation in the d/L ratio with the number of panels for Pratt parallel-chord trusses. In the range of trusses investigated, each of these lines is only slightly curved and the entire group lies close to the straight line corresponding to the equation

$$d/L = 0.381 - 0.0185n \quad (7)$$

In fact, the use of this equation to determine the depth of truss results in a maximum deviation in the weight of truss of less than 0.1 percent

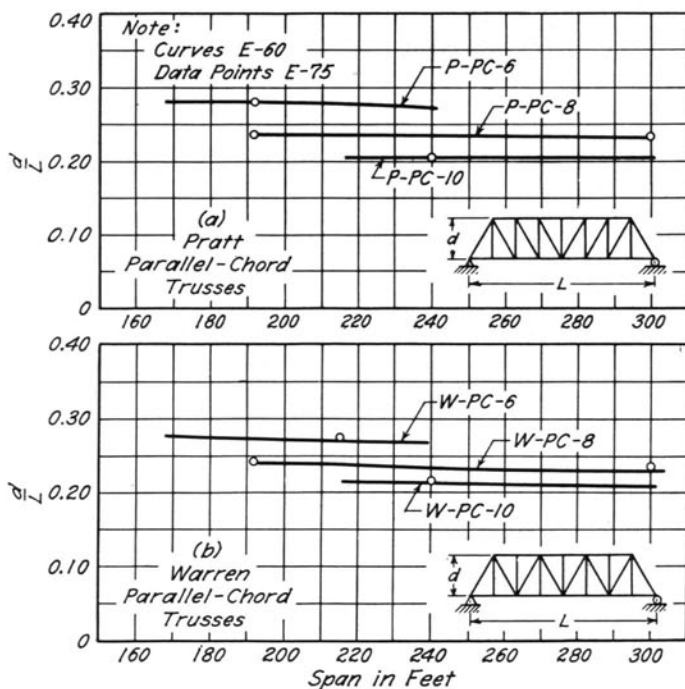
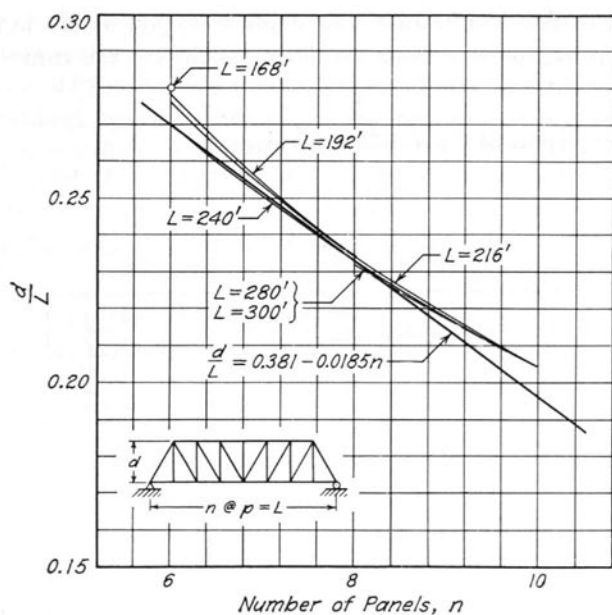
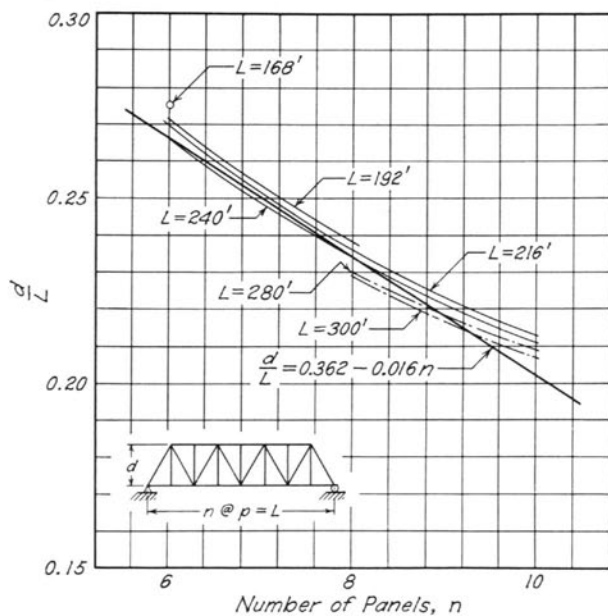


Fig. 7. Ratios of Least-Weight Depth to Span for Parallel-Chord Trusses

from the least weight for the range of trusses investigated. The straight line approximation was placed to give depths less than (if different from) the calculated values of least-weight depths in the range of span lengths which would normally call for a parallel-chord truss. Figure 9 is a summary of similar results for Warren parallel-chord trusses. The straight-line approximation

$$d/L = 0.362 - 0.016n \quad (8)$$


 Fig. 8. Variation in d/L Ratio with Number of Panels for Pratt Parallel-Chord Trusses

 Fig. 9. Variation in d/L Ratio with Number of Panels for Warren Parallel-Chord Trusses

for the Warren trusses results in a maximum deviation in the weight of truss of less than 0.2 percent from the least weight for the range represented by these calculations.

16. Least-Weight Depths of Curved-Chord Trusses

Results of a similar nature to those summarized for parallel-chord trusses are obtained for curved-chord trusses. Figure 10 shows the influence of span length on the ratios of least-weight depths to span length

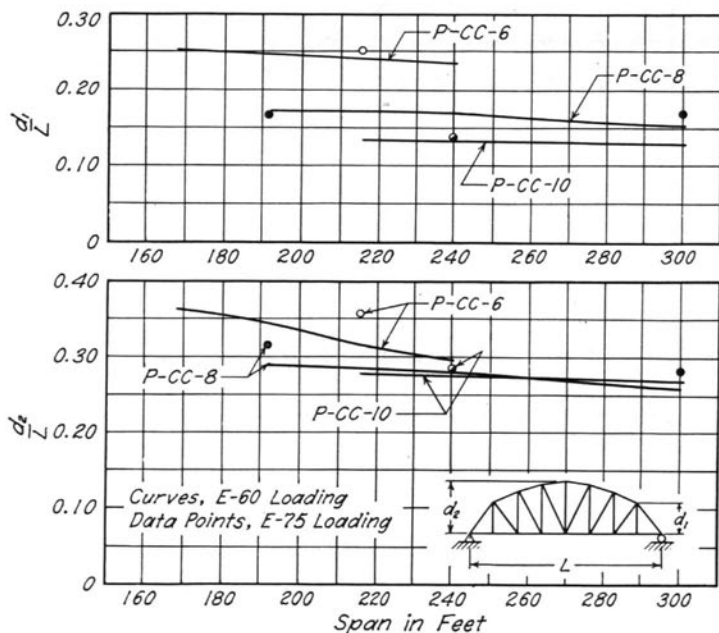


Fig. 10. Ratios of Least-Weight Depths to Span for Pratt Curved-Chord Trusses

for Pratt curved-chord trusses. The two ratios d_1/L and d_2/L are plotted for each truss investigated. The least-weight ratios d_1/L and d_2/L for curved-chord trusses cannot be approximated by simple formulas as accurately as for the parallel-chord trusses. However, in the range of trusses investigated and for span lengths for which curved-chord trusses are commonly used (say, for convenience, greater than 216 ft) the following pair of formulas represents a fair approximation to the least-weight proportions of Pratt curved-chord trusses:

$$d_1 = L \left(-0.187 + 0.0105n + \frac{2.12}{n} \right) \quad (9a)$$

$$d_2 = 0.27 L \quad (9b)$$

Figure 11 shows the relation of the first of these formulas to the calculated data while the second formula represents a straight horizontal line at $d_2/L = 0.27$ on the lower curve of Fig. 10. Obviously the greatest error is introduced by the second formula applied to 6-panel trusses. The weight of a 216-ft, 6-panel Pratt truss proportioned according to these two formulas is greater than the least weight possible by about 3.5 percent. The greatest weight increase due to the use of these formulas for the Pratt trusses of 8 and 10 panels is 0.2 percent.

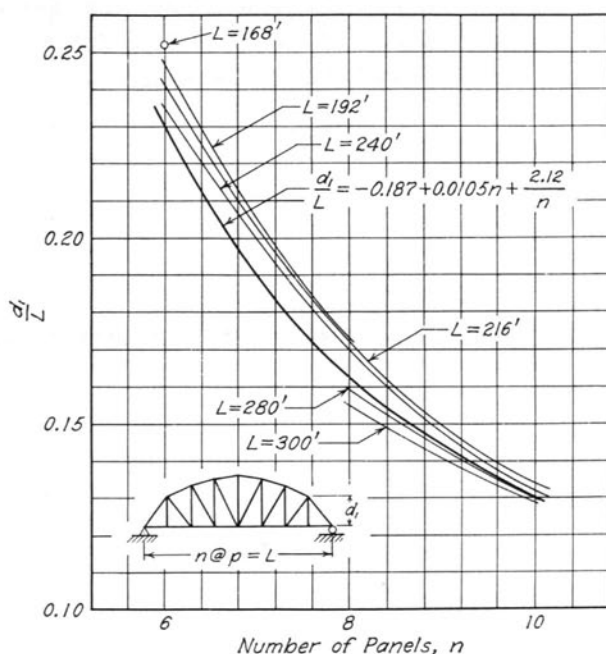


Fig. 11. Variation in d_1/L Ratio with Number of Panels for Pratt Curved-Chord Trusses

Figures 12 and 13 are similar to Figs. 10 and 11, but they are the result of calculations for Warren curved-chord trusses. A study of these data reveals that the two equations

$$d_1 = L (0.350 - 0.0225n) \quad (10a)$$

$$d_2 = 0.25L \quad (10b)$$

give a fair approximation to least-weight proportions of Warren trusses having 6, 8, or 10 panels and spans of 216 to 300 ft. In this range of trusses, the greatest weight increase over least weight due to the use of

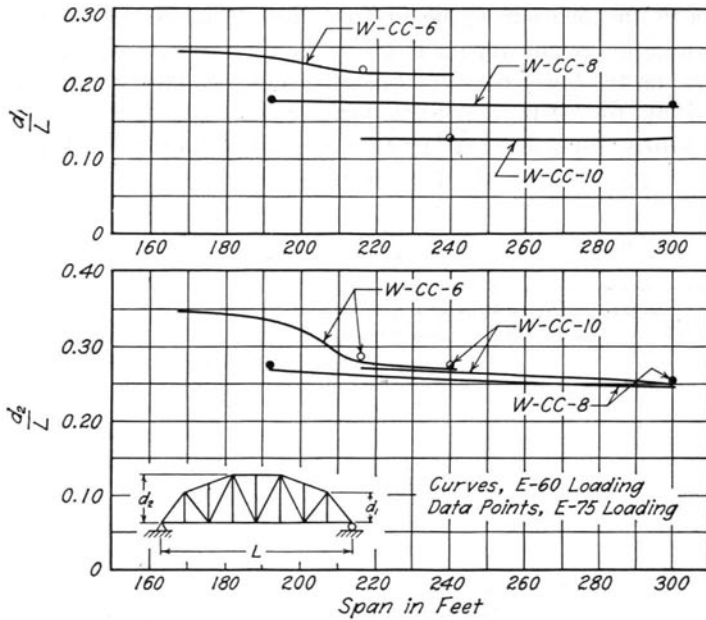


Fig. 12. Ratios of Least-Weight Depths to Span for Warren Curved-Chord Trusses

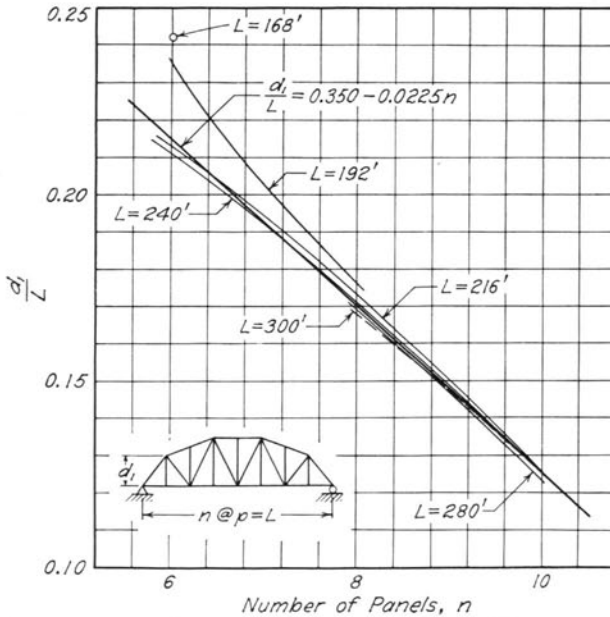


Fig. 13. Variation in d_1/L Ratio with Number of Panels for Warren Curved-Chord Trusses

this pair of approximate formulas is about 3.0 percent for 6-panel trusses and 0.4 percent or less for 8- and 10-panel trusses. One should be cautious about extrapolating values for the first of these two Warren truss formulas beyond the range of the available data. An increase in the number of panels of the Warren truss to twelve or more would probably require an approximation curve having some similarity in form to Eq. 9a rather than Eq. 10a above.

17. Least Weights of Trusses

The results of the calculation of least weight of each truss investigated are shown on Figs. 14 and 15 for the parallel- and curved-chord trusses, respectively. For convenience in subsequent calculations, the relative efficiency of the truss material is indicated in the graphs by the ratios w_{tb}/w_{etb} which are plotted as ordinates on span lengths as abscissas. In order to compare these theoretically-determined least weights with weights obtained in practice, the dashed curves on Figs. 14 and 15 are shown to represent data published by J. A. L. Waddell.⁽¹⁹⁾ The practical data were obtained by Waddell from weight records of bridges actually designed and built. He made no distinction between parallel- and curved-chord trusses nor between bridges with different web systems. He indicated in his publication only that the information is applicable to simple-truss, double-track, carbon steel railway bridges. In comparing the theoretical and practical results, one should keep in mind the ranges of span lengths in which parallel- and curved-chord trusses are normally built. If one chooses to say that parallel-chord trusses may be used for spans of 216 ft or shorter and that curved-chord trusses may be used for spans of 192 ft or longer, the theoretical curves for these chosen ranges all lie under the practical curve. This is to be expected since the theoretical weights are the least weights possible for the trusses that are indicated on the curves.

An interesting comparison of weights of comparable Pratt and Warren parallel-chord trusses is made by Fig. 14. The two types are equal in weight for 6-panel trusses, while for 8 and 10 panels the Pratt trusses are heavier by 6 to 7 and 9 to 11 percent respectively than the corresponding Warren trusses. These data indicate that the difference in weight increases with an increase in number of panels.

Similar weight comparisons are made in Fig. 15 for the curved-chord trusses. Pratt curved-chord trusses of 6 panels range from the same in weight to about 8 percent lighter than the corresponding Warren trusses. The least weights obtainable with 8- and 10-panel trusses are practically equal for the two types of web systems.

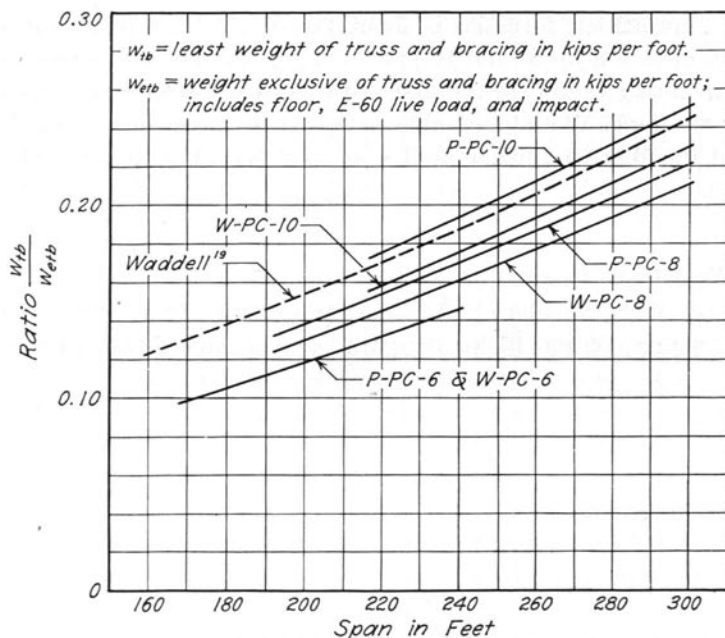
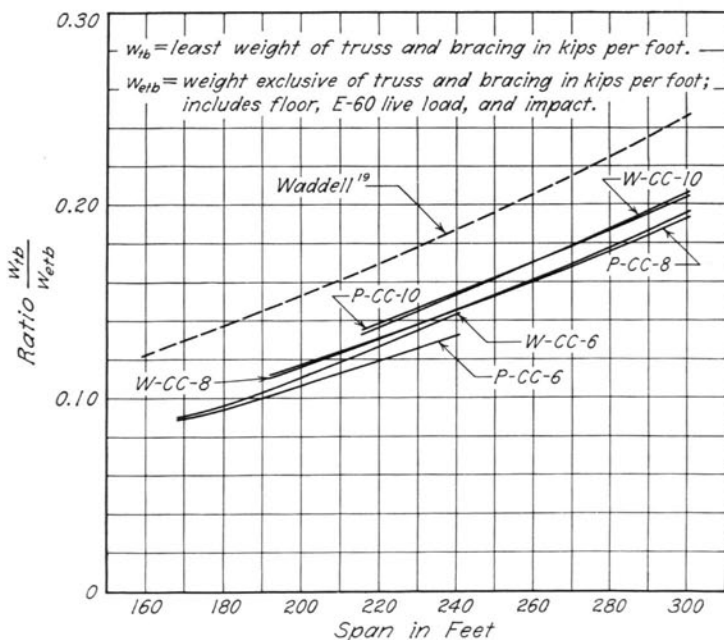
Fig. 14. Variation of Ratio w_{lb}/w_{eib} with Span for Parallel-Chord TrussesFig. 15. Variation of Ratio w_{lb}/w_{eib} with Span for Curved-Chord Trusses

Figure 16 is presented to indicate the general influence of the number of panels, in given span lengths, upon the least dead load of bridge supported by each truss. The dead load consists of one-half of the floor weight combined with the weight of truss and bracing. The dashed portions of the curves are estimated extensions of the results obtained from the calculations. These two span lengths—216 ft and 240 ft—are the only ones for which the calculations include as many as three different

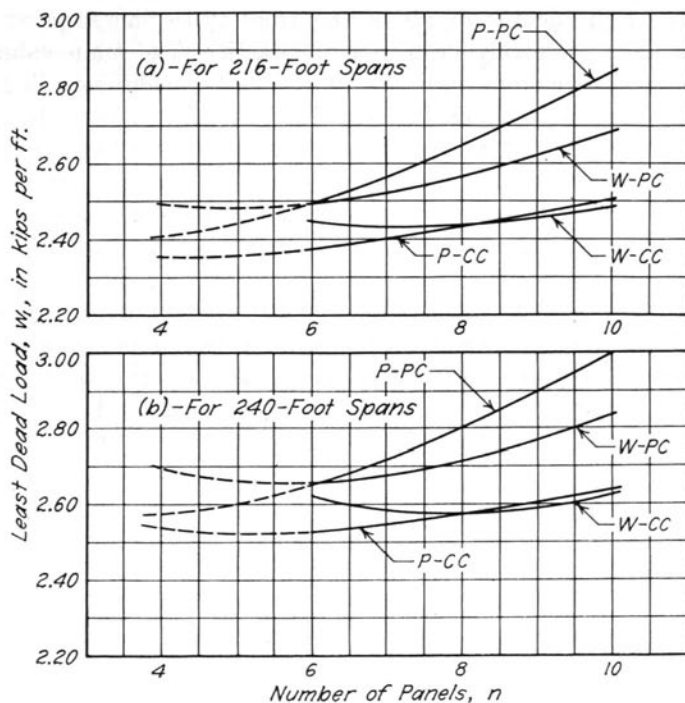


Fig. 16. Influence of Number of Panels on Least Dead Load

panel lengths, and the curves can be useful only to indicate trends in the over-all influence of the number of panels in a given span.

18. Practical Significance of Results

This study results in confirmation of the general trends stated by Jackson⁽¹³⁾ for Warren parallel-chord trusses and those being obtained by Shoukry for Warren curved-chord trusses. In addition, it establishes similar trends for Pratt parallel- and curved-chord trusses.

In general, the least-weight depths determined for trusses similar to those of Jackson are slightly greater than the depths which he determined

by design procedure. This may be due in part to difficulty in choosing the exact minimum point of a plotted volume curve and in part to inaccuracies in the assumptions made in this theoretical approach. That the differences are not great is indicated by the fact that in every case common to the two studies, the range of depths determined by Jackson as giving weight of truss differing by one percent or less from the least weight includes the least-weight depth which is determined by this theoretical study.

The least-weight depths of all of the truss types investigated are greater than have commonly been used in practice. The main value of these results may stem from estimates which can be made from them to point the way toward economical design. For example, suppose the bare members of a truss of a given type and a given depth have been designed and the weight of the members has been determined. Adding about 37.5 percent to the weight of the bare members gives an approximation to the weight of truss including details. Twelve and one-half percent of the latter weight can then be taken as the approximate weight of bracing. The combined weight of truss and bracing and the design depths can then be compared with the theoretical least-weight characteristics obtained in this study by plotting two points on a weight-depth curve as shown in Fig 17. Point *A* represents the preliminary design of the truss and point *B* represents the least-weight design. Assuming the curve between *A* and *B* is approximately a parabola with its axis of symmetry vertical and its vertex located at the minimum point, one can estimate easily the weight saving which would result from a given change in depth. If more accurate information on the rate of change in weight with respect to a change in depth is desired, one can use the derivatives formulated during the calculations. For a large change in depth, the volume formulas can be used to determine the resulting volume or weight of metal. Such studies of weight change would facilitate the determination of the best proportions for the outline of the truss; their use could result in the design of somewhat deeper trusses in future erections.

In making use of the results of this investigation, it is to be remembered that these calculations are based upon and satisfy modern design specifications, which in turn are based to a large extent upon past and present design practice. Design of railway bridge trusses having depths as great as those indicated may introduce problems not fully covered by present specifications. For example, vibrations in long web members of deep trusses may not be adequately controlled by the present specifications governing slenderness ratios of these members. This factor remains to be investigated.

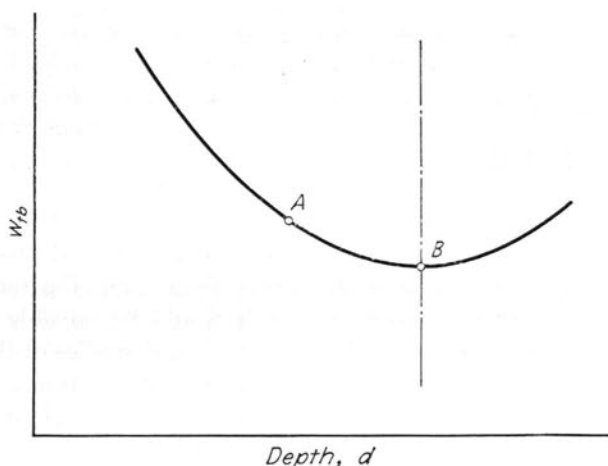


Fig. 17. Weight-Depth Curve

The least-weight depths of the trusses investigated are somewhat greater and the resulting truss weights are somewhat less than are commonly encountered in present designs. Since the design of the members making up the lateral bracing between trusses is usually governed by slenderness ratio requirements rather than by stress requirements (especially in double-track bridges), the 12.5 percent of truss weight which is assumed for the weight of the lateral bracing may be slightly less than would be required for the deeper trusses. However, the least-weight depths of these trusses are influenced very little by changes in weight or loading; this can be seen by comparing the least-weight depths obtained for E-60 and E-75 live loading on the various trusses. Therefore any reasonable increase in the weight of lateral bracing would have negligible effect upon the least-weight proportions of the trusses investigated in this study.

IV. CONCLUSIONS

As a result of this research, the following conclusions can be drawn:

(1) It is entirely feasible to apply basic mathematical procedures to the determination of least-weight proportions and theoretical least weights of bridge trusses. Furthermore, it would be possible to extend this procedure — after further individual statistical studies of the relative costs of material, rolling, fabrication, erection, and maintenance of the various types of members of trusses as well as a study of the influence of truss depths upon cost of bracing — to determine most economical proportions of bridge trusses.

(2) Least-weight depths of through-type, double-track, open-timber-deck, Pratt parallel-chord trusses designed according to AREA specifications for E-60 live load are given approximately by the formula

$$d = L (0.381 - 0.0185n) \quad \begin{matrix} (168 \leq L \leq 300) \\ (6 \leq n \leq 10) \end{matrix} \quad (7)$$

(3) Least-weight depths of through-type, double-track, open-timber-deck, Warren parallel-chord trusses designed according to AREA specifications for E-60 live load are given approximately by the formula

$$d = L (0.362 - 0.016n) \quad \begin{matrix} (168 \leq L \leq 300) \\ (6 \leq n \leq 10) \end{matrix} \quad (8)$$

(4) Least-weight depths of through-type, double-track, open-timber-deck, Pratt curved-chord trusses designed according to AREA specifications for E-60 live load are given approximately by the formulas

$$\begin{matrix} d_1 = L \left(-0.187 + 0.0105n + \frac{2.12}{n} \right) \\ d_2 = 0.27 L \end{matrix} \left\{ \begin{matrix} (216 \leq L \leq 300) \\ (6 \leq n \leq 10) \end{matrix} \right. \quad (9)$$

(5) Least-weight depths of through-type, double-track, open-timber-deck, Warren curved-chord trusses designed according to AREA specifications for E-60 live load are given approximately by the formulas

$$\begin{matrix} d_1 = L (0.350 - 0.0225n) \\ d_2 = 0.25 L \end{matrix} \left\{ \begin{matrix} (216 \leq L \leq 300) \\ (6 \leq n \leq 10) \end{matrix} \right. \quad (10)$$

(6) Least-weight depths of parallel-chord bridge trusses designed for E-75 live load are about one to two percent greater than for corresponding trusses designed for E-60 live load.

(7) Least-weight depths of curved-chord bridge trusses designed for E-75 live load average about five and one-half percent (ranging from three to nine percent) greater than for corresponding trusses designed for E-60 live load.

(8) Pratt and Warren parallel-chord trusses of 6 panels are almost identical in least weight. At 8 and 10 panels, Pratt trusses are heavier by 6 to 7 and by 9 to 11 percent respectively than corresponding Warren trusses.

(9) Warren curved-chord trusses of 6 panels range from the same in weight to 8 percent heavier than corresponding Pratt trusses. The least weights of 8- and 10-panel Warren trusses are practically equal to those of the corresponding Pratt trusses.

The conclusions which are drawn from the results of this study will prove most valuable to the engineer who understands their nature and who employs them properly. Even at the risk of unnecessary repetition, a reminder concerning the use for which these results are intended seems appropriate.

The concluding generalizations are intended not as a substitute for but as an aid to engineering judgment. Since engineering design constantly entails compromises of some sort between best design and least cost, and since least weight of structure is but one factor in least cost, an engineer cannot correctly use as design formulas the approximate equations or the more exact graphical data pertinent to least-weight proportions. However, one who knows the least-weight characteristics of a structure is much more able to make a judicious compromise between best design and least cost than one who must guess at this information.

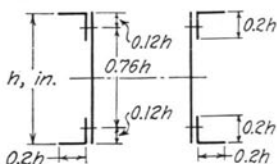
V. BIBLIOGRAPHY

1. Whipple, Squire, *Bridge Building*, Utica, N.Y., H. H. Curtiss, 1847.
2. Pratt, T. Willis, "Some Observations on Trusses and Trussed Arches," *Transactions of ASCE*, vol. I, p. 346, 1872.
3. Adler, Emil, "Note on the Most Economical Depth of Straight Girders and Trusses," *Van Nostrand's Engineering Magazine*, vol. XVI, p. 42, 1877.
4. Emery, Charles E., "Relative Quantities of Material in Bridges of Different Kinds, of Various Heights," *Transactions of ASCE*, vol. VI, pp. 235 and 277, 1877.
5. Searles, William E., "Discussion of 'Relative Quantities of Material in Bridges of Different Kinds, of Various Heights' by C. E. Emery," *Transactions of ASCE*, vol. VII, p. 192, 1878.
6. Du Bois, A. J., "Formulas for the Weights of Bridges," *Transactions of ASCE*, vol. XVI, p. 191, 1887.
7. Lundie, John, "Economical Height of Bridge Trusses for a Given Panel Width," *Journal of Association of Engineering Societies*, vol. VII, p. 101, 1887.
8. Waddell, J. A. L., "True Economy in the Design of Bridges," *Engineering News*, vol. XXXIV, p. 179, 1895.
9. Melan, J., "Fachwerktrager Mit Weiter Ausfachung," *Der Eisenbau*, vol. 5, p. 272, 1914.
10. Angst, Maximilian H., "Untersuchungen über die Trägergewichte und die günstigsten Trägerabmessungen bei Parallel- und Halbparabelträgern mit untenliegender Fahrbahn," (Dissertation) Zurich, 1915.
11. Voigt, Adolf, "Die günstigsten Höhen der als Strebenfachwerke Ausgebildeten Hauptträger eiserner Eisenbahnbrücken," *Die Bautechnik*, vol. 9, p. 768, 1931.
12. Erdmann, Otfried, "Über den Einfluss hochfester Stähle auf Gewichtsparsnis und Bauart im Stahlbrückenbau," *Forschungshefte aus dem Gebiete des Stahlbaues*, Herausgegeben vom Fachverband Stahlbau, Deutscher Stahlbau-Verband, Bad Pyrmont, Berlin/Göttingen/Heidelberg, Springer-Verlag, 1950.
13. Jackson, Melvin W., "A Study of the Economic Proportions of Steel Truss Bridges" (Dissertation), University of Illinois, 1948.
14. "Specifications for Steel Railway Bridges for Fixed Spans Not Exceeding 400 Feet in Length," American Railway Engineering Association, Chicago 5, Illinois, 1947.
15. Sherwood, G. E. F. and Taylor, A. E., *Calculus*, pp. 456-458, New York, Prentice Hall, Inc., 1947.
16. Shedd, T. C., *Structural Design in Steel*, pp. 476 and 479, New York, John Wiley and Sons, 1934.
17. Kunz, F. C., *Design of Steel Bridges*, p. 224, New York, McGraw-Hill Book Company, 1915.
18. Waddell, J. A. L., *Bridge Engineering*, pp. 1239 and 1240, New York, John Wiley and Sons, 1925.
19. Waddell, J. A. L., "Weights of Metal in Steel Trusses," *Transactions of ASCE*, vol. 101, p. 1, 1936.

APPENDIX A: SAMPLE DERIVATIONS OF GENERAL VOLUME FORMULAS

Volume of Members Governed by Slenderness Ratio

Assume the general form and proportions of cross section as shown: i.e., four angle bars and two web plates.



According to AREA⁽¹⁴⁾ specifications:

Part IV, sec. 406, paragraph (b),

$$\text{Minimum thickness of angles} = \frac{1}{12} \times 0.2h = 0.0167h \text{ (in.)}$$

Part IV, sec. 405,

$$\text{Minimum thickness of web plates} = \frac{1}{40}^* \times 0.76h = 0.019h \text{ (in.)}$$

then,

$$\begin{aligned} \text{Minimum area of section} &\cong 8 \times 0.2h \times 0.0167h = 0.0267h^2 \text{ (in.}^2\text{)} \\ &+ 2h \times 0.019h = 0.0380h^2 \\ &\cong 0.0647h^2 \text{ (in.}^2\text{)} \end{aligned}$$

For approximate radius of gyration, see *Bridge Engineering*, p. 504, by J. A. L. Waddell:

$$r \cong 0.36h$$

then

$$\frac{l}{r} = \frac{l \times 12}{0.36h}$$

If maximum $\frac{l}{r}$ allowable = 100, then $100 = \frac{12l}{0.36h}$ and

$$h \text{ (in.)} = \frac{l}{3} \text{ (ft).}$$

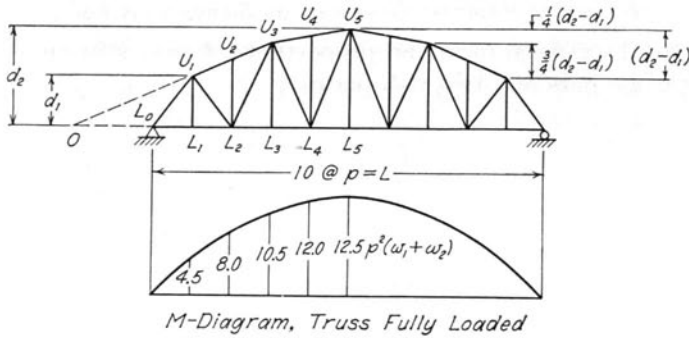
* AREA specifies $\frac{1}{32}$ for web plates of segments and $\frac{1}{40}$ for cover plates or web plates connecting segments. In this investigation the latter ratio was used to determine minimum thickness of web plates.

$$\text{Minimum area of section} = 0.0647h^2 = 0.0647 \frac{l^2}{9} = 0.0072l^2$$

$$\text{Minimum volume of member} = 0.0072l^3 \text{ (in.}^2\text{-ft)}$$

Likewise, if maximum $\frac{l}{r} = 120$, minimum volume = $0.0050l^3$ (in.²-ft).

Derivation of Volume Formulas for Representative Members of W-CC-10



Lengths:

$$U_1-L_1 = d_1$$

$$U_2-L_2 = \frac{1}{8} (5d_1 + 3d_2)$$

$$U_3-L_3 = \frac{1}{4} (d_1 + 3d_2)$$

$$U_4-L_4 = \frac{1}{8} (d_1 + 7d_2)$$

$$U_5-L_5 = d_2$$

$$O-L_0 = \frac{p}{3} \frac{11d_1 - 3d_2}{d_2 - d_1}$$

$$O-L_2 = \frac{p}{3} \frac{5d_1 + 3d_2}{d_2 - d_1}$$

$$O-L_3 = \frac{p}{3} \frac{2d_1 + 6d_2}{d_2 - d_1}$$

MEMBER L_2-L_4 :

$$\text{Maximum stress} = \frac{10.5p^2 (w_1 + w_2)}{\frac{1}{4} (d_1 + 3d_2)} = \frac{42p^2 (w_1 + w_2)}{d_1 + 3d_2}$$

$$\text{Area required} = \frac{42p^2 (w_1 + w_2)}{T (d_1 + 3d_2)}$$

$$\text{Volume} = \frac{42p^2 (w_1 + w_2)}{T (d_1 + 3d_2)} 2p = \frac{84p^3}{T} \frac{(w_1 + w_2)}{(d_1 + 3d_2)}$$

MEMBER U_1-U_3 :

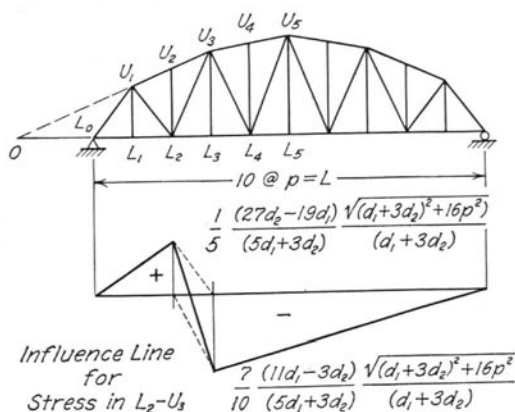
$$\text{Maximum stress} = \frac{8.0p^2 (w_1 + w_2)}{\frac{1}{8} (5d_1 + 3d_2)} \frac{\sqrt{\frac{9}{16} (d_2 - d_1)^2 + 4p^2}}{2p}$$

$$\text{Area required} = \frac{32p}{C} \frac{(w_1 + w_2) \sqrt{\frac{9}{16} (d_2 - d_1)^2 + 4p^2}}{(5d_1 + 3d_2)}$$

$$\text{Volume} = \frac{32p}{C} \frac{(w_1 + w_2) \left[\frac{9}{16} (d_2 - d_1)^2 + 4p^2 \right]}{(5d_1 + 3d_2)}$$

$$\text{Volume} = \frac{2p}{C} \frac{(w_1 + w_2) [9 (d_2 - d_1)^2 + 64p^2]}{5d_1 + 3d_2}$$

MEMBER L₂-U₃:



$$\begin{aligned} \text{DL stress}^{\text{(tens.)}} &= \left[\frac{3}{2} p w_1 \times \frac{1}{5} (27 d_2 - 19 d_1) - 4 p w_1 \times \frac{7}{10} (11 d_1 - 3 d_2) \right] \\ &\times \frac{\sqrt{(d_1 + 3 d_2)^2 + 16 p^2}}{(5 d_1 + 3 d_2) (d_1 + 3 d_2)} \\ &= p w_1 \frac{(16.5 d_2 - 36.5 d_1) \sqrt{(d_1 + 3 d_2)^2 + 16 p^2}}{(5 d_1 + 3 d_2) (d_1 + 3 d_2)} \end{aligned}$$

$$\begin{aligned} \text{LL stress (tens.)} &= \frac{3}{2} p w_2 \times \frac{1}{5} \frac{(27d_2 - 19d_1) \sqrt{(d_1 + 3d_2)^2 + 16p^2}}{(5d_1 + 3d_2)(d_1 + 3d_2)} \\ &= p w_2 \frac{(8.1d_2 - 5.7d_1) \sqrt{(d_1 + 3d_2)^2 + 16p^2}}{(5d_1 + 3d_2)(d_1 + 3d_2)} \end{aligned}$$

$$\begin{aligned}\text{LL stress (comp.)} &= 4pw_2 \times \frac{7}{10} \frac{(11d_1 - 3d_2) \sqrt{(d_1 + 3d_2)^2 + 16p^2}}{(5d_1 + 3d_2)(d_1 + 3d_2)} \\ &= pw_2 \frac{(30.8d_1 - 8.4d_2) \sqrt{(d_1 + 3d_2)^2 + 16p^2}}{(5d_1 + 3d_2)(d_1 + 3d_2)}\end{aligned}$$

Assuming reversal of stress with tension predominate:

$$S_{\max} (\text{tens.}) = \text{DL stress (tens.)} + \text{LL stress (tens.)}$$

$$S_{\min} (\text{comp.}) = -\text{DL stress (tens.)} + \text{LL stress (comp.)}$$

$$S_{D \max} (\text{tens.}) = S_{\max} (\text{tens.}) + \frac{1}{2} S_{\min} (\text{comp.})$$

$$\begin{aligned}&= \frac{1}{2} \text{DL stress (tens.)} + \text{LL stress (tens.)} \\ &\quad + \frac{1}{2} \text{LL stress (comp.)} \\ &= \frac{pw_1}{2} \frac{(16.5d_2 - 36.5d_1) \sqrt{(d_1 + 3d_2)^2 + 16p^2}}{(5d_1 + 3d_2)(d_1 + 3d_2)} \\ &\quad + pw_2 \frac{(3.9d_2 + 9.7d_1) \sqrt{(d_1 + 3d_2)^2 + 16p^2}}{(5d_1 + 3d_2)(d_1 + 3d_2)}\end{aligned}$$

$$\begin{aligned}S_{D \min} (\text{comp.}) &= \frac{3}{2} S_{\min} (\text{comp.}) \\ &= \frac{3pw_1}{2} \frac{(36.5d_1 - 16.5d_2) \sqrt{(d_1 + 3d_2)^2 + 16p^2}}{(5d_1 + 3d_2)(d_1 + 3d_2)} \\ &\quad + \frac{3pw_2}{2} \frac{(30.8d_1 - 8.4d_2) \sqrt{(d_1 + 3d_2)^2 + 16p^2}}{(5d_1 + 3d_2)(d_1 + 3d_2)}\end{aligned}$$

$$\begin{aligned}V_{(t)} &= \frac{pw_1}{8T} \frac{(16.5d_2 - 36.5d_1) [(d_1 + 3d_2)^2 + 16p^2]}{(5d_1 + 3d_2)(d_1 + 3d_2)} \\ &\quad + \frac{pw_2}{4T} \frac{(9.7d_1 + 3.9d_2) [(d_1 + 3d_2)^2 + 16p^2]}{(5d_1 + 3d_2)(d_1 + 3d_2)}\end{aligned}$$

$$\text{or for } \frac{l}{r} = 120, V_{(\frac{l}{r})} = 0.0050l^3 = 7.80 \times 10^{-5} [(d_1 + 3d_2)^2 + 16p^2]^{3/2}$$

$$\begin{aligned}V_{(c)} &= \frac{3pw_1}{8C} \frac{(36.5d_1 - 16.5d_2) [(d_1 + 3d_2)^2 + 16p^2]}{(5d_1 + 3d_2)(d_1 + 3d_2)} \\ &\quad + \frac{3pw_2}{8C} \frac{(30.8d_1 - 8.4d_2) [(d_1 + 3d_2)^2 + 16p^2]}{(5d_1 + 3d_2)(d_1 + 3d_2)}\end{aligned}$$

$$\text{or for } \frac{l}{r} = 100, V_{(\frac{l}{r})} = 0.0072l^3 = 11.232 \times 10^{-5} [(d_1 + 3d_2)^2 + 16p^2]^{3/2}$$

or, assuming reversal of stress with compression predominate:

$$S_{\max} (\text{comp.}) = \text{DL stress (comp.)} + \text{LL stress (comp.)}$$

$$S_{\min} (\text{tens.}) = -\text{DL stress (comp.)} + \text{LL stress (tens.)}$$

$$S_{D \max} (\text{comp.}) = S_{\max} (\text{comp.}) + \frac{1}{2} S_{\min} (\text{tens.})$$

$$= \frac{1}{2} \text{DL stress (comp.)} + \text{LL stress (comp.)}$$

$$+ \frac{1}{2} \text{LL stress (tens.)}$$

$$= \frac{pw_1}{2} \frac{(36.5d_1 - 16.5d_2) \sqrt{(d_1 + 3d_2)^2 + 16p^2}}{(5d_1 + 3d_2)(d_1 + 3d_2)}$$

$$+ pw_2 \frac{(27.95d_1 - 4.35d_2) \sqrt{(d_1 + 3d_2)^2 + 16p^2}}{(5d_1 + 3d_2)(d_1 + 3d_2)}$$

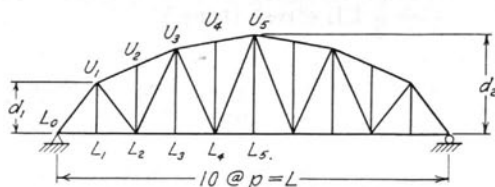
$$V_{(c)} = \frac{pw_1}{8C} \frac{(36.5d_1 - 16.5d_2) [(d_1 + 3d_2)^2 + 16p^2]}{(5d_1 + 3d_2)(d_1 + 3d_2)}$$

$$+ \frac{pw_2}{4C} \frac{(27.95d_1 - 4.35d_2) [(d_1 + 3d_2)^2 + 16p^2]}{(5d_1 + 3d_2)(d_1 + 3d_2)}$$

$$\text{or, for } \frac{l}{r} = 100, V_{(\frac{l}{r})} = 0.0072l^3 = 11.232 \times 10^{-5} [(d_1 + 3d_2)^2 + 16p^2]^{3/2}$$

APPENDIX B: GENERAL VOLUME FORMULAS AND DERIVATIVE EQUATIONS FOR WARREN TEN-PANEL CURVED-CHORD TRUSSES

General Volume Formulas for W-CC-10



Member	V
L_0-L_2	$\frac{9p^3}{T} \frac{(w_1 + w_2)}{d_1}$
L_2-L_4	$\frac{84p^3}{T} \frac{(w_1 + w_2)}{d_1 + 3d_2}$
L_4-L_5	$\frac{12.5p^3}{T} \frac{(w_1 + w_2)}{d_2}$
U_1-U_3	$\frac{2p}{C} (w_1 + w_2) \frac{9(d_2 - d_1)^2 + 64p^2}{5d_1 + 3d_2}$
U_3-U_5	$\frac{3p}{C} (w_1 + w_2) \frac{(d_2 - d_1)^2 + 64p^2}{d_1 + 7d_2}$
L_0-U_1	$\frac{4.5p}{C} (w_1 + w_2) \frac{d_1^2 + p^2}{d_1}$
U_1-L_1	$\frac{p}{T} (w_1 + w_2) d_1$
U_3-L_3	$\frac{p}{4T} (w_1 + w_2) (d_1 + 3d_2)$ or $2.8125 \times 10^{-5} (d_1 + 3d_2)^3$

U_5-L_5	$\frac{p}{2T} (w_1 + w_2) d_2$ $\text{or } 9.0 \times 10^{-4} d_2^3$
U_1-L_2	$\frac{pw_1}{T} \frac{(41.5d_1 - 13.5d_2) (d_1^2 + p^2)}{d_1 (5d_1 + 3d_2)}$ $+ \frac{pw_2}{T} \frac{(39.6d_1 - 10.8d_2) (d_1^2 + p^2)}{d_1 (5d_1 + 3d_2)}$
L_2-U_3	$\frac{pw_1}{8T} \frac{(16.5d_2 - 36.5d_1) [(d_1 + 3d_2)^2 + 16p^2]}{(5d_1 + 3d_2) (d_1 + 3d_2)}$ $+ \frac{pw_2}{4T} \frac{(9.7d_1 + 3.9d_2) [(d_1 + 3d_2)^2 + 16p^2]}{(5d_1 + 3d_2) (d_1 + 3d_2)}$ $\text{or } 7.80 \times 10^{-5} [(d_1 + 3d_2)^2 + 16p^2]^{3/2}$ <hr/> $\text{or } \frac{3pw_1}{8C} \frac{(36.5d_1 - 16.5d_2) [(d_1 + 3d_2)^2 + 16p^2]}{(5d_1 + 3d_2) (d_1 + 3d_2)}$ $+ \frac{3pw_2}{8C} \frac{(30.8d_1 - 8.4d_2) [(d_1 + 3d_2)^2 + 16p^2]}{(5d_1 + 3d_2) (d_1 + 3d_2)}$ $\text{or } 11.232 \times 10^{-5} [(d_1 + 3d_2)^2 + 16p^2]^{3/2}$ <hr/> $\text{or } \frac{pw_1}{8C} \frac{(36.5d_1 - 16.5d_2) [(d_1 + 3d_2)^2 + 16p^2]}{(5d_1 + 3d_2) (d_1 + 3d_2)}$ $+ \frac{pw_2}{4C} \frac{(27.95d_1 - 4.35d_2) [(d_1 + 3d_2)^2 + 16p^2]}{(5d_1 + 3d_2) (d_1 + 3d_2)}$ $\text{or } 11.232 \times 10^{-5} [(d_1 + 3d_2)^2 + 16p^2]^{3/2}$
U_3-L_4	$\frac{pw_1}{8T} \frac{(13.5d_1 - 1.5d_2) [(d_1 + 3d_2)^2 + 16p^2]}{(d_1 + 7d_2) (d_1 + 3d_2)}$ $+ \frac{pw_2}{4T} \frac{(9d_1 + 10.2d_2) [(d_1 + 3d_2)^2 + 16p^2]}{(d_1 + 7d_2) (d_1 + 3d_2)}$ $\text{or } 7.80 \times 10^{-5} [(d_1 + 3d_2)^2 + 16p^2]^{3/2}$ <hr/> $\text{or } \frac{pw_1}{4C} \frac{(2.25d_2 - 20.25d_1) [(d_1 + 3d_2)^2 + 16p^2]}{(d_1 + 7d_2) (d_1 + 3d_2)}$ $+ \frac{pw_2}{4C} \frac{(11.7d_2 - 4.5d_1) [(d_1 + 3d_2)^2 + 16p^2]}{(d_1 + 7d_2) (d_1 + 3d_2)}$ $\text{or } 11.232 \times 10^{-5} [(d_1 + 3d_2)^2 + 16p^2]^{3/2}$

L_4-U_5	$\frac{pw_1}{2T} \frac{(8.5d_2 - 12.5d_1)(d_2^2 + p^2)}{d_2(d_1 + 7d_2)}$ $+ \frac{pw_2}{T} \frac{(15.25d_2 - 1.25d_1)(d_2^2 + p^2)}{d_2(d_1 + 7d_2)}$ $\text{or } 0.0050 (d_2^2 + p^2)^{3/2}$ <hr/> $\text{or } \frac{3pw_1}{2C} \frac{(12.5d_1 - 8.5d_2)(d_2^2 + p^2)}{d_2(d_1 + 7d_2)}$ $+ \frac{3pw_2}{2C} \frac{(7.5d_1 + 4.5d_2)(d_2^2 + p^2)}{d_2(d_1 + 7d_2)}$ $\text{or } 0.0072 (d_2^2 + p^2)^{3/2}$
U_2-L_2	$9.75 \times 10^{-6} (5d_1 + 3d_2)^3$
U_4-L_4	$9.75 \times 10^{-6} (d_1 + 7d_2)^3$

Derivative Equation No. 1 for W-CC-10

Mem-ber	$\frac{\partial V}{\partial d_1}$
L_0-L_2	$-\frac{9p^3}{T} \frac{(w_1 + w_2)}{d_1^2}$
L_2-L_4	$-\frac{84p^3}{T} \frac{(w_1 + w_2)}{(d_1 + 3d_2)^2}$
L_4-L_5	0
U_1-U_3	$\frac{2p}{C} (w_1 + w_2) \left\{ \frac{-18(d_2 - d_1)}{5d_1 + 3d_2} - \frac{45(d_2 - d_1)^2 + 320p^2}{(5d_1 + 3d_2)^2} \right\}$
U_3-U_5	$\frac{3p}{C} (w_1 + w_2) \left\{ \frac{-2(d_2 - d_1)}{d_1 + 7d_2} - \frac{(d_2 - d_1)^2 + 64p^2}{(d_1 + 7d_2)^2} \right\}$
L_0-U_1	$\frac{4.5p}{C} (w_1 + w_2) \left(1 - \frac{p^2}{d_1^2} \right)$
U_1-L_1	$\frac{p}{T} (w_1 + w_2)$

U_3-L_3	$\frac{p}{4T} (w_1 + w_2)$ $\text{or } 8.4375 \times 10^{-5} (d_1 + 3d_2)^2$
U_5-L_5	0 $\text{or } 0$
U_1-L_2	$\frac{pw_1}{T} \left\{ \frac{41.5 (d_1^2 + p^2) + (41.5d_1 - 13.5d_2) 2d_1}{d_1 (5d_1 + 3d_2)} \right.$ $\left. - \frac{(41.5d_1 - 13.5d_2) (d_1^2 + p^2) (10d_1 + 3d_2)}{d_1^2 (5d_1 + 3d_2)^2} \right\}$ $+ \frac{pw_2}{T} \left\{ \frac{39.6 (d_1^2 + p^2) + (39.6d_1 - 10.8d_2) 2d_1}{d_1 (5d_1 + 3d_2)} \right.$ $\left. - \frac{(39.6d_1 - 10.8d_2) (d_1^2 + p^2) (10d_1 + 3d_2)}{d_1^2 (5d_1 + 3d_2)^2} \right\}$
L_2-U_3	$\frac{pw_1}{8T} \left\{ \frac{-36.5 [(d_1 + 3d_2)^2 + 16p^2] + (16.5d_2 - 36.5d_1) 2(d_1 + 3d_2)}{(5d_1 + 3d_2) (d_1 + 3d_2)} \right.$ $\left. - \frac{(16.5d_2 - 36.5d_1) [(d_1 + 3d_2)^2 + 16p^2] (10d_1 + 18d_2)}{(5d_1 + 3d_2)^2 (d_1 + 3d_2)^2} \right\}$ $+ \frac{pw_2}{4T} \left\{ \frac{9.7 [(d_1 + 3d_2)^2 + 16p^2] + (9.7d_1 + 3.9d_2) 2(d_1 + 3d_2)}{(5d_1 + 3d_2) (d_1 + 3d_2)} \right.$ $\left. - \frac{(9.7d_1 + 3.9d_2) [(d_1 + 3d_2)^2 + 16p^2] (10d_1 + 18d_2)}{(5d_1 + 3d_2)^2 (d_1 + 3d_2)^2} \right\}$ $\text{or } 2.34 \times 10^{-4} [(d_1 + 3d_2)^2 + 16p^2]^{1/2}$
or	$\frac{3pw_1}{8C} \left\{ \frac{36.5 [(d_1 + 3d_2)^2 + 16p^2] + (36.5d_1 - 16.5d_2) 2(d_1 + 3d_2)}{(5d_1 + 3d_2) (d_1 + 3d_2)} \right.$ $\left. - \frac{(36.5d_1 - 16.5d_2) [(d_1 + 3d_2)^2 + 16p^2] (10d_1 + 18d_2)}{(5d_1 + 3d_2)^2 (d_1 + 3d_2)^2} \right\}$ $+ \frac{3pw_2}{8C} \left\{ \frac{30.8 [(d_1 + 3d_2)^2 + 16p^2] + (30.8d_1 - 8.4d_2) 2(d_1 + 3d_2)}{(5d_1 + 3d_2) (d_1 + 3d_2)} \right.$ $\left. - \frac{(30.8d_1 - 8.4d_2) [(d_1 + 3d_2)^2 + 16p^2] (10d_1 + 18d_2)}{(5d_1 + 3d_2)^2 (d_1 + 3d_2)^2} \right\}$ $\text{or } 3.3696 \times 10^{-4} (d_1 + 3d_2) [(d_1 + 3d_2)^2 + 16p^2]^{1/2}$

or

$$\begin{aligned} & \frac{pw_1}{8C} \left\{ \frac{36.5 [(d_1+3d_2)^2+16p^2] + (36.5d_1-16.5d_2) 2(d_1+3d_2)}{(5d_1+3d_2)(d_1+3d_2)} \right. \\ & \quad \left. - \frac{(36.5d_1-16.5d_2) [(d_1+3d_2)^2+16p^2] (10d_1+18d_2)}{(5d_1+3d_2)^2 (d_1+3d_2)^2} \right\} \\ & + \frac{pw_2}{4C} \left\{ \frac{27.95 [(d_1+3d_2)^2+16p^2] + (27.95d_1-4.35d_2) 2(d_1+3d_2)}{(5d_1+3d_2)(d_1+3d_2)} \right. \\ & \quad \left. - \frac{(27.95d_1-4.35d_2) [(d_1+3d_2)^2+16p^2] (10d_1+18d_2)}{(5d_1+3d_2)^2 (d_1+3d_2)^2} \right\} \\ & \text{or } 3.3696 \times 10^{-4} (d_1+3d_2) [(d_1+3d_2)^2+16p^2]^{1/2} \end{aligned}$$

 U_3-L_4

$$\begin{aligned} & \frac{pw_1}{8T} \left\{ \frac{13.5 [(d_1+3d_2)^2+16p^2] + (13.5d_1-1.5d_2) 2(d_1+3d_2)}{(d_1+7d_2)(d_1+3d_2)} \right. \\ & \quad \left. - \frac{(13.5d_1-1.5d_2) [(d_1+3d_2)^2+16p^2] (2d_1+10d_2)}{(d_1+7d_2)^2 (d_1+3d_2)^2} \right\} \\ & + \frac{pw_2}{4T} \left\{ \frac{9 [(d_1+3d_2)^2+16p^2] + (9d_1+10.2d_2) 2(d_1+3d_2)}{(d_1+7d_2)(d_1+3d_2)} \right. \\ & \quad \left. - \frac{(9d_1+10.2d_2) [(d_1+3d_2)^2+16p^2] (2d_1+10d_2)}{(d_1+7d_2)^2 (d_1+3d_2)^2} \right\} \\ & \text{or } 2.34 \times 10^{-4} (d_1+3d_2) [(d_1+3d_2)^2+16p^2]^{1/2} \end{aligned}$$

or

$$\begin{aligned} & \frac{pw_1}{4C} \left\{ \frac{-20.25 [(d_1+3d_2)^2+16p^2] + (2.25d_2-20.25d_1) 2(d_1+3d_2)}{(d_1+7d_2)(d_1+3d_2)} \right. \\ & \quad \left. - \frac{(2.25d_2-20.25d_1) [(d_1+3d_2)^2+16p^2] (2d_1+10d_2)}{(d_1+7d_2)^2 (d_1+3d_2)^2} \right\} \\ & + \frac{pw_2}{4C} \left\{ \frac{-4.5 [(d_1+3d_2)^2+16p^2] + (11.7d_2-4.5d_1) 2(d_1+3d_2)}{(d_1+7d_2)(d_1+3d_2)} \right. \\ & \quad \left. - \frac{(11.7d_2-4.5d_1) [(d_1+3d_2)^2+16p^2] (2d_1+10d_2)}{(d_1+7d_2)^2 (d_1+3d_2)^2} \right\} \\ & \text{or } 3.3696 \times 10^{-4} (d_1+3d_2) [(d_1+3d_2)^2+16p^2]^{1/2} \end{aligned}$$

 L_4-U_5

$$\begin{aligned} & \frac{pw_1}{2T} \left\{ \frac{-12.5 (d_2^2+p^2)}{d_2 (d_1+7d_2)} - \frac{(8.5d_2-12.5d_1) (d_2^2+p^2)}{d_2 (d_1+7d_2)^2} \right\} \\ & + \frac{pw_2}{T} \left\{ \frac{-1.25 (d_2^2+p^2)}{d_2 (d_1+7d_2)} - \frac{(15.25d_2-1.25d_1) (d_2^2+p^2)}{d_2 (d_1+7d_2)^2} \right\} \\ & \text{or } 0 \end{aligned}$$

	or $\frac{3pw_1}{2C} \left\{ \frac{12.5 (d_2^2 + p^2)}{d_2 (d_1 + 7d_2)} - \frac{(12.5d_1 - 8.5d_2) (d_2^2 + p^2)}{d_2 (d_1 + 7d_2)^2} \right\}$ $+ \frac{3pw_2}{2C} \left\{ \frac{7.5 (d_2^2 + p^2)}{d_2 (d_1 + 7d_2)} - \frac{(7.5d_1 + 4.5d_2) (d_2^2 + p^2)}{d_2 (d_1 + 7d_2)^2} \right\}$ or 0
U_2-L_2	$1.4625 \times 10^{-4} (5d_1 + 3d_2)^2$
U_4-L_4	$2.925 \times 10^{-5} (d_1 + 7d_2)^2$
	Eq. 1: $\sum \frac{\partial V}{\partial d_1} = 0$

Derivative Equation No. 2 for W-CC-10

Mem-ber	$\frac{\partial V}{\partial d_2}$
L_0-L_2	0
L_2-L_4	$-\frac{252p^3}{T} \frac{(w_1 + w_2)}{(d_1 + 3d_2)^2}$
L_4-L_5	$-\frac{12.5p^3}{T} \frac{(w_1 + w_2)}{d_2^2}$
U_1-U_3	$\frac{2p}{C} (w_1 + w_2) \left\{ \frac{18 (d_2 - d_1)}{5d_1 + 3d_2} - \frac{27 (d_2 - d_1)^2 + 192p^2}{(5d_1 + 3d_2)^2} \right\}$
U_3-U_5	$\frac{3p}{C} (w_1 + w_2) \left\{ \frac{2 (d_2 - d_1)}{d_1 + 7d_2} - \frac{7 (d_2 - d_1)^2 + 448p^2}{(d_1 + 7d_2)^2} \right\}$
L_0-U_1	0
U_1-L_1	0
U_3-L_3	$\frac{3p}{4T} (w_1 + w_2)$ or $25.3125 \times 10^{-5} (d_1 + 3d_2)^2$
U_5-L_5	$\frac{p}{2T} (w_1 + w_2)$ or $27.0 \times 10^{-4} d_2^2$
U_1-L_2	$\frac{pw_1}{T} \left\{ \frac{-13.5 (d_1^2 + p^2)}{d_1 (5d_1 + 3d_2)} - \frac{(41.5d_1 - 13.5d_2) (d_1^2 + p^2)3}{d_1 (5d_1 + 3d_2)^2} \right\}$

$$\begin{aligned}
& + \frac{pw_2}{T} \left\{ \frac{-10.8 (d_1^2 + p^2)}{d_1 (5d_1 + 3d_2)} - \frac{(39.6d_1 - 10.8d_2) (d_1^2 + p^2) 3}{d_1 (5d_1 + 3d_2)^2} \right\} \\
\hline
L_2-U_3 & \frac{pw_1}{8T} \left\{ \frac{16.5 [(d_1 + 3d_2)^2 + 16p^2] + (16.5d_2 - 36.5d_1) 6(d_1 + 3d_2)}{(5d_1 + 3d_2) (d_1 + 3d_2)} \right. \\
& \quad \left. - \frac{(16.5d_2 - 36.5d_1) [(d_1 + 3d_2)^2 + 16p^2] (18d_1 + 18d_2)}{(5d_1 + 3d_2)^2 (d_1 + 3d_2)^2} \right\} \\
& + \frac{pw_2}{4T} \left\{ \frac{3.9 [(d_1 + 3d_2)^2 + 16p^2] + (9.7d_1 + 3.9d_2) 6(d_1 + 3d_2)}{(5d_1 + 3d_2) (d_1 + 3d_2)} \right. \\
& \quad \left. - \frac{(9.7d_1 + 3.9d_2) [(d_1 + 3d_2)^2 + 16p^2] (18d_1 + 18d_2)}{(5d_1 + 3d_2)^2 (d_1 + 3d_2)^2} \right\} \\
& \quad \text{or } 7.02 \times 10^{-4} (d_1 + 3d_2) [(d_1 + 3d_2)^2 + 16p^2]^{1/2} \\
& \quad \text{or} \\
& \frac{3pw_1}{8C} \left\{ \frac{-16.5 [(d_1 + 3d_2)^2 + 16p^2] + (36.5d_1 - 16.5d_2) 6(d_1 + 3d_2)}{(5d_1 + 3d_2) (d_1 + 3d_2)} \right. \\
& \quad \left. - \frac{(36.5d_1 - 16.5d_2) [(d_1 + 3d_2)^2 + 16p^2] (18d_1 + 18d_2)}{(5d_1 + 3d_2)^2 (d_1 + 3d_2)^2} \right\} \\
& + \frac{3pw_2}{8C} \left\{ \frac{-8.4 [(d_1 + 3d_2)^2 + 16p^2] + (30.8d_1 - 8.4d_2) 6(d_1 + 3d_2)}{(5d_1 + 3d_2) (d_1 + 3d_2)} \right. \\
& \quad \left. - \frac{(30.8d_1 - 8.4d_2) [(d_1 + 3d_2)^2 + 16p^2] (18d_1 + 18d_2)}{(5d_1 + 3d_2)^2 (d_1 + 3d_2)^2} \right\} \\
& \quad \text{or } 1.011 \times 10^{-3} (d_1 + 3d_2) [(d_1 + 3d_2)^2 + 16p^2]^{1/2} \\
& \quad \text{or} \\
& \frac{pw_1}{8C} \left\{ \frac{-16.5 [(d_1 + 3d_2)^2 + 16p^2] + (36.5d_1 - 16.5d_2) 6(d_1 + 3d_2)}{(5d_1 + 3d_2) (d_1 + 3d_2)} \right. \\
& \quad \left. - \frac{(36.5d_1 - 16.5d_2) [(d_1 + 3d_2)^2 + 16p^2] (18d_1 + 18d_2)}{(5d_1 + 3d_2)^2 (d_1 + 3d_2)^2} \right\} \\
& + \frac{pw_2}{4C} \left\{ \frac{-4.35 [(d_1 + 3d_2)^2 + 16p^2] + (27.95d_1 - 4.35d_2) 6(d_1 + 3d_2)}{(5d_1 + 3d_2) (d_1 + 3d_2)} \right. \\
& \quad \left. - \frac{(27.95d_1 - 4.35d_2) [(d_1 + 3d_2)^2 + 16p^2] (18d_1 + 18d_2)}{(5d_1 + 3d_2)^2 (d_1 + 3d_2)^2} \right\} \\
& \quad \text{or } 1.011 \times 10^{-3} (d_1 + 3d_2) [(d_1 + 3d_2)^2 + 16p^2]^{1/2}
\end{aligned}$$

U_3-L_4	$\frac{pw_1}{8T} \left\{ \frac{-1.5 [(d_1+3d_2)^2+16p^2] + (13.5d_1-1.5d_2) 6(d_1+3d_2)}{(d_1+7d_2) (d_1+3d_2)} \right.$ $\left. - \frac{(13.5d_1-1.5d_2) [(d_1+3d_2)^2+16p^2] (10d_1+42d_2)}{(d_1+7d_2)^2 (d_1+3d_2)^2} \right\}$ $+ \frac{pw_2}{4T} \left\{ \frac{10.2 [(d_1+3d_2)^2+16p^2] + (9d_1+10.2d_2) 6(d_1+3d_2)}{(d_1+7d_2) (d_1+3d_2)} \right.$ $\left. - \frac{(9d_1+10.2d_2) [(d_1+3d_2)^2+16p^2] (10d_1+42d_2)}{(d_1+7d_2)^2 (d_1+3d_2)^2} \right\}$ $\text{or } 7.02 \times 10^{-4} (d_1+3d_2) [(d_1+3d_2)^2+16p^2]^{1/2}$ <hr/> or $\frac{pw_1}{4C} \left\{ \frac{2.25 [(d_1+3d_2)^2+16p^2] + (2.25d_2-20.25d_1) 6(d_1+3d_2)}{(d_1+7d_2) (d_1+3d_2)} \right.$ $\left. - \frac{(2.25d_2-20.25d_1) [(d_1+3d_2)^2+16p^2] (10d_1+42d_2)}{(d_1+7d_2)^2 (d_1+3d_2)^2} \right\}$ $+ \frac{pw_2}{4C} \left\{ \frac{11.7 [(d_1+3d_2)^2+16p^2] + (11.7d_2-4.5d_1) 6(d_1+3d_2)}{(d_1+7d_2) (d_1+3d_2)} \right.$ $\left. - \frac{(11.7d_2-4.5d_1) [(d_1+3d_2)^2+16p^2] (10d_1+42d_2)}{(d_1+7d_2)^2 (d_1+3d_2)^2} \right\}$ $\text{or } 1.011 \times 10^{-3} (d_1+3d_2) [(d_1+3d_2)^2+16p^2]^{1/2}$ <hr/>
L_4-U_5	$\frac{pw_1}{2T} \left\{ \frac{8.5 (d_2^2 + p^2) + (8.5d_2 - 12.5d_1) 2d_2}{d_2 (d_1 + 7d_2)} \right.$ $\left. - \frac{(8.5d_2 - 12.5d_1) (d_2^2 + p^2) (d_1 + 14d_2)}{d_2^2 (d_1 + 7d_2)^2} \right\}$ $+ \frac{pw_2}{T} \left\{ \frac{15.25 (d_2^2 + p^2) + (15.25d_2 - 1.25d_1) 2d_2}{d_2 (d_1 + 7d_2)} \right.$ $\left. - \frac{(15.25d_2 - 1.25d_1) (d_2^2 + p^2) (d_1 + 14d_2)}{d_2^2 (d_1 + 7d_2)^2} \right\}$ $\text{or } 0.0150d_2 (d_2^2 + p^2)^{1/2}$

or

$$\begin{aligned}
& \frac{3pw_1}{2C} \left\{ \frac{-8.5 (d_2^2 + p^2) + (12.5d_1 - 8.5d_2) 2d_2}{d_2 (d_1 + 7d_2)} \right. \\
& \quad \left. - \frac{(12.5d_1 - 8.5d_2) (d_2^2 + p^2) (d_1 + 14d_2)}{d_2^2 (d_1 + 7d_2)^2} \right\} \\
& + \frac{3pw_2}{2C} \left\{ \frac{4.5 (d_2^2 + p^2) + (7.5d_1 + 4.5d_2) 2d_2}{d_2 (d_1 + 7d_2)} \right. \\
& \quad \left. - \frac{(7.5d_1 + 4.5d_2) (d_2^2 + p^2) (d_1 + 14d_2)}{d_2^2 (d_1 + 7d_2)^2} \right\} \\
& \text{or } 0.0216d_2 (d_2^2 + p^2)^{1/2}
\end{aligned}$$

$$U_2-L_2 \quad 8.775 \times 10^{-5} (5d_1 + 3d_2)^2$$

$$U_4-L_4 \quad 2.0475 \times 10^{-4} (d_1 + 7d_2)^2$$

$$\text{Eq. 2: } \sum \frac{\partial V}{\partial d_2} = 0$$

The Engineering Experiment Station was established by act of the University of Illinois Board of Trustees on December 8, 1903. Its purpose is to conduct engineering investigations that are important to the industrial interests of the state.

The management of the Station is vested in an Executive Staff composed of the Director, the Associate Director, the heads of the departments in the College of Engineering, the professor in charge of Chemical Engineering, and the director of Engineering Information and Publications. This staff is responsible for establishing the general policies governing the work of the Station. All members of the College of Engineering teaching staff are encouraged to engage in the scientific research of the Station.

To make the results of its investigations available to the public, the Station publishes a series of bulletins. Occasionally it publishes circulars which may contain timely information compiled from various sources not readily accessible to the Station clientele or may contain important information obtained during the investigation of a particular research project but not having a direct bearing on it. A few reprints of articles appearing in the technical press and written by members of the staff are also published.

In ordering copies of these publications reference should be made to the Engineering Experiment Station Bulletin, Circular, or Reprint Series number which is just above the title on the cover. Address

THE ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS

